Prep (2) : Second Term (2012 – 2013) : Geometry Rules

Theorem (1) :

Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are equal in area.

Corollary (1)

The parallelogram and the rectangle with common base and between two parallel straight lines are equal in area.

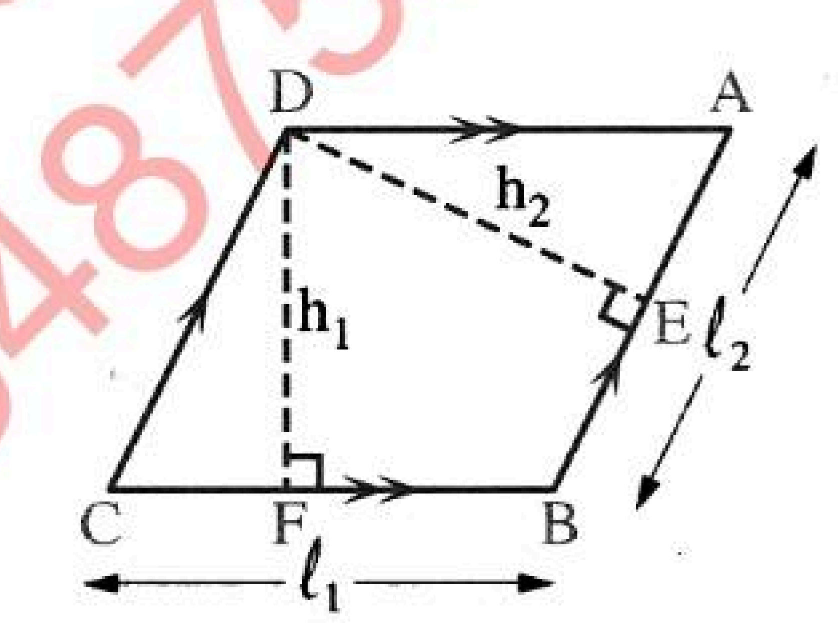
Corollary (2)

The area of the parallelogram = the length of the base x its corresponding height.
In the opposite figure :

Remark :

In the opposite figure :

If ABCD is a parallelogram , DF is the corresponding height of the base $\overline{BC}$ and DE is the corresponding height of the base $\overline{AB}$, then : The area of the parallelogram ABCD = BC × DF = AB × DE



i.e. $\ell_1 \times h_1 = \ell_2 \times h_2$

Corollary (3)

The parallelograms with bases equal in length and lying on a straight line , while the opposite sides to these bases are on another straight line , are equal in area.

Corollary (4) :

Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.

Corollary (5)

Area of the triangle = $\frac{1}{2}$ of the length of the base × its corresponding height

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Theorem (2)

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.

Corollary (1)

Triangles of bases equal in length and lying between two parallel straight lines are equal in area.

Corollary (2)

The median of a triangle divides its surface into two triangular surfaces equal in area.

Corollary (3)

Triangles with congruent bases on one straight line and have a common vertex are equal in areas.

Theorem (3)

If two triangles are equal in area and drawn on the same base and on one side of it , then their vertices lie on a straight line parallel to this base.

Remark

If two triangles have the same area and they are included between two straight lines and their bases on these two straight lines are equal in length , then the two straight lines are parallel.

Unit (5) : Lesson (1) :Similarity

Definition

It is said that the two polygons P_1 and P_2 (of the same number of sides) are similar if the following two conditions are verified together :

1. Their corresponding angles are equal in measure.
2. The corresponding side lengths are proportional.

In this case , we write the polygon $P_1 \sim$ the polygon P_2

That means the polygon P_1 is similar to the polygon P_2

Remark :

A geometric fact :

The two triangles are similar if one of the two following conditions is verified :

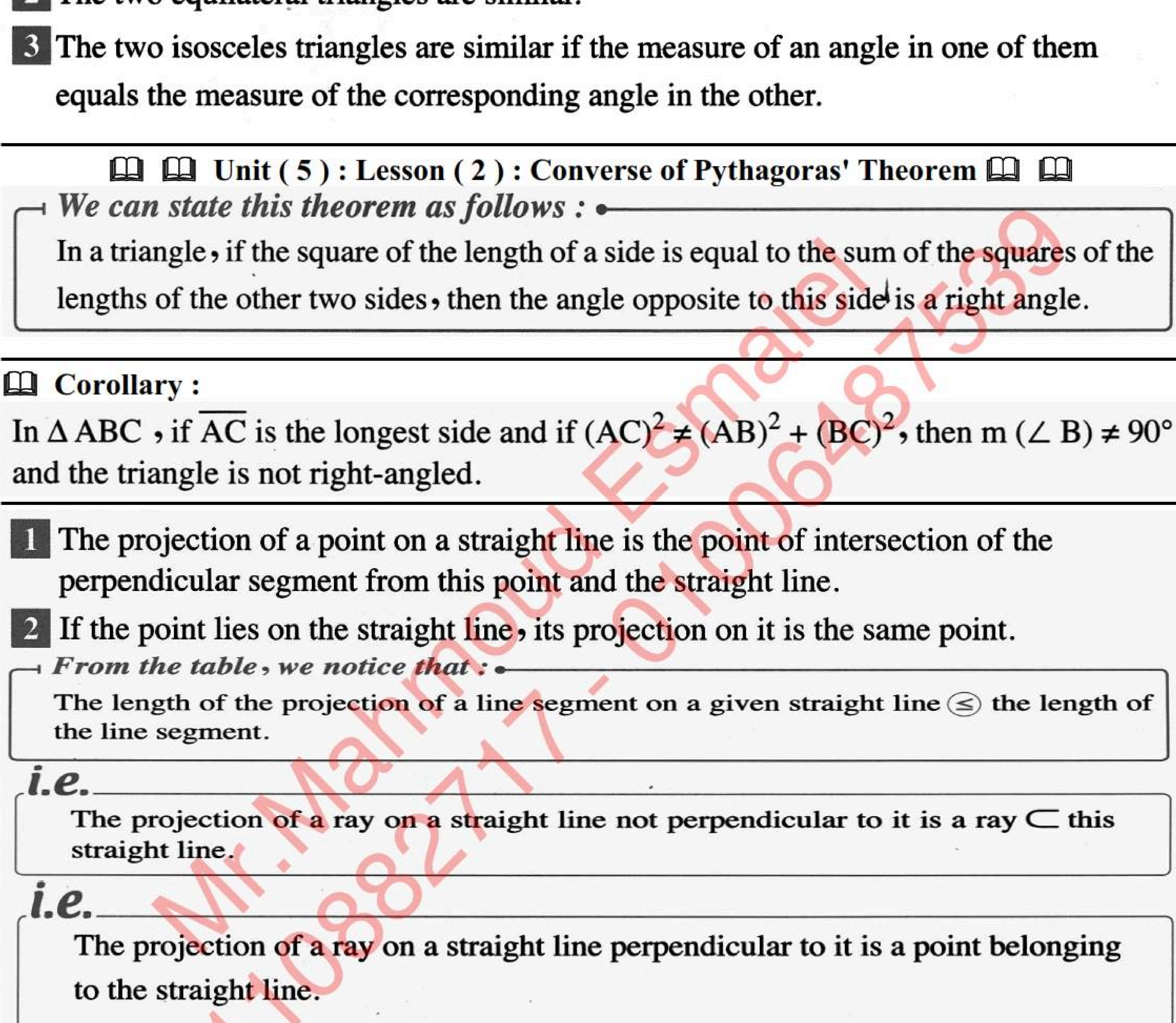
1. The measures of their corresponding angles are equal.
2. The lengths of their corresponding sides are proportional.

Remark :

1 The two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other.

2 The two equilateral triangles are similar.

3 The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.



Unit (5) : Lesson (2) : Converse of Pythagoras' Theorem

We can state this theorem as follows :

In a triangle, if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

Corollary :

In ΔABC, if $\overline{AC}$ is the longest side and if $(AC)^2 \neq (AB)^2 + (BC)^2$, then $m(\angle B) \neq 90°$ and the triangle is not right-angled.

1 The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point and the straight line.

2 If the point lies on the straight line, its projection on it is the same point.

From the table, we notice that :

The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

i.e.

The projection of a ray on a straight line not perpendicular to it is a ray $\subset$ this straight line.

i.e.

The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.

i.e.

The projection of a straight line on a straight line not perpendicular to it is a straight line.

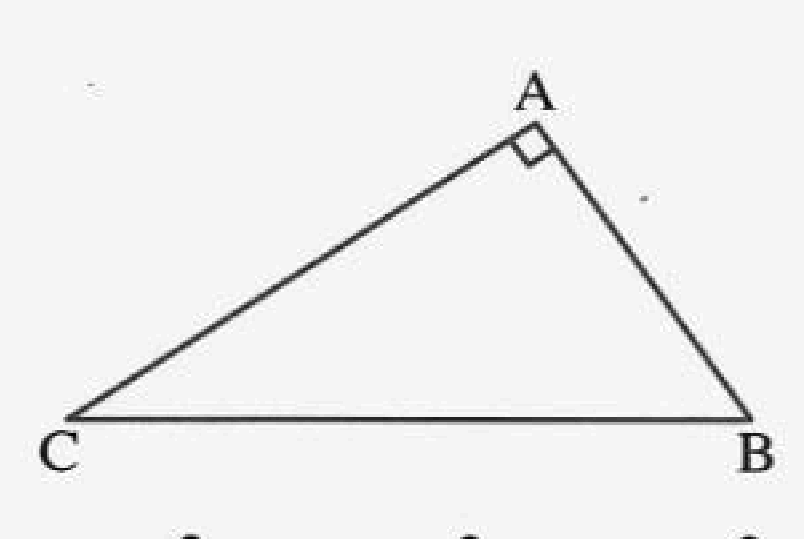
i.e.

The projection of a straight line on a straight line perpendicular to it is the point of intersection of the two straight lines.

Unit (5) : Lesson (4) : Euclidean Theorem

In the right-angled triangle , the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

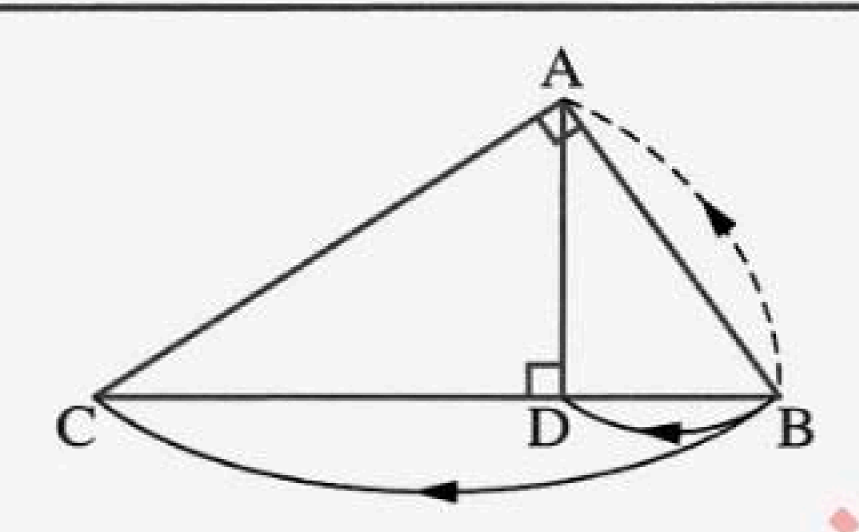
In the following , we write the summary of the relations of Pythagoras' theorem and Euclidean theorem :



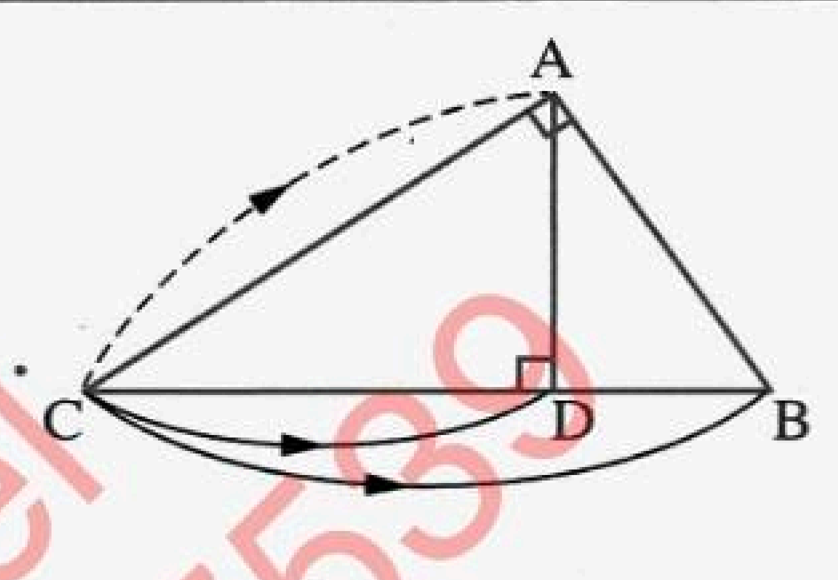
$(BC)^2 = (AB)^2 + (AC)^2$

$(AB)^2 = (BC)^2 - (AC)^2$

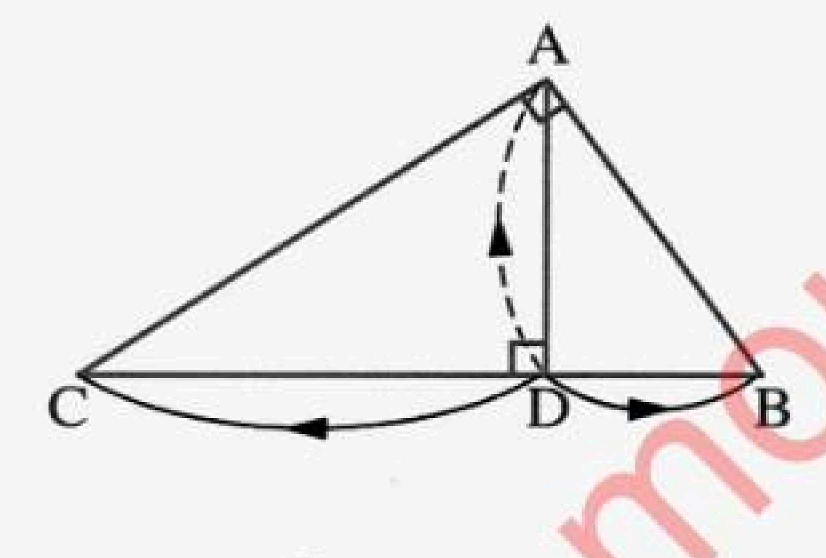
$(AC)^2 = (BC)^2 - (AB)^2$



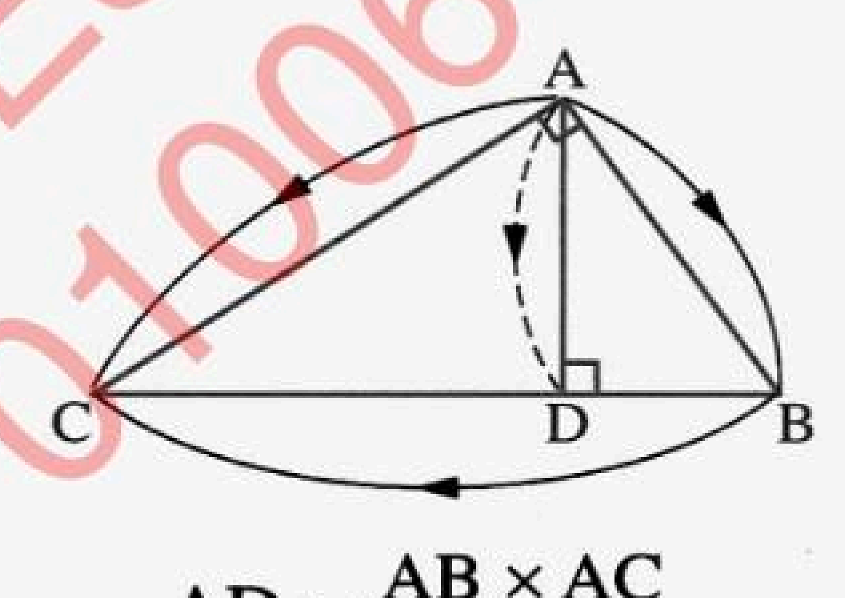
$(BA)^2 = BD \times BC$



$(CA)^2 = CD \times CB$



$(DA)^2 = DB \times DC$



$AD = \frac{AB \times AC}{BC}$

Unit (5) : Lesson (5) : Classifying triangles according to their angles

If the square length of the longest side equals the sum of the squares lengths of the other two sides , then the triangle is right-angled.

i.e.

If the square length of the longest side is greater than the sum of squares lengths of the other two sides , then the triangle is obtuse-angled.

i.e.

If the square length of the longest side is less than the sum of squares lengths of the other two sides , then the triangle is acute-angled.

1. To determine the type of an angle in a triangle , we compare between the square length of the side opposite to it and the sum of squares lengths of the other two sides.
2. The greatest angle in measure in the triangle is opposite to the longest side.
3. In any triangle , there are two acute angles at least.

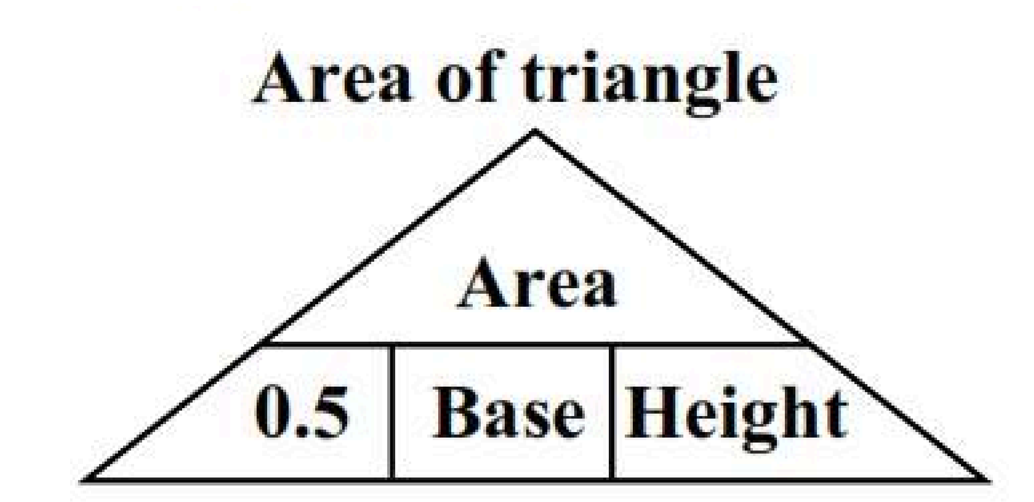
☏ ☏ All on laws Primary Stage ☏ ☏

Triangle : Area = 0.5 × Base × Height

: Perimeter = SL1 + SL2 + SL3

: Perimeter of equilateral = SL × 3

Area of triangle



Rectangle : Area = L × W

: Perimeter = (L + W) × 2

: L = Area ÷ W = 0.5 × P – W

: W = Area ÷ L = 0.5 × P – L

Area of Rectangle



Square : : Area = L × L = 0.5 × D × D

D : Diagonal = $\sqrt{Area \times 2}$

: Perimeter = L × 4

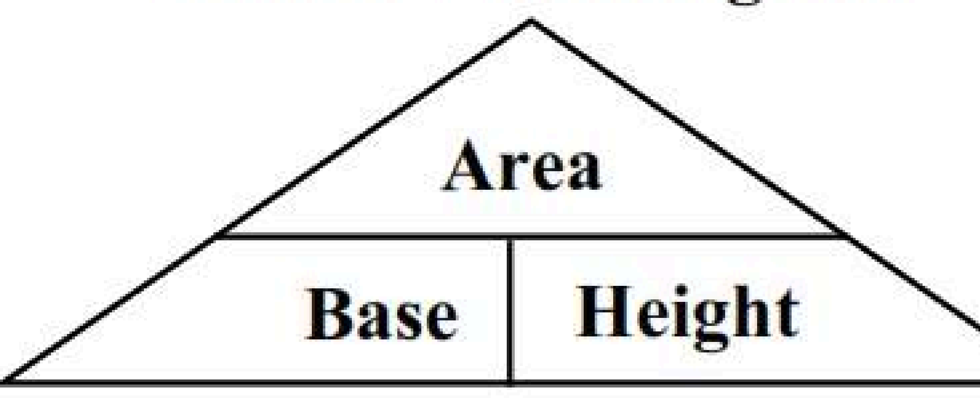
: L = $\sqrt{Area}$ **= Perimeter ÷ 4**

Parallelogram : Area = Base × Height

: Base = Area ÷ Height

: Height = Area ÷ Base

Area of Parallelogram

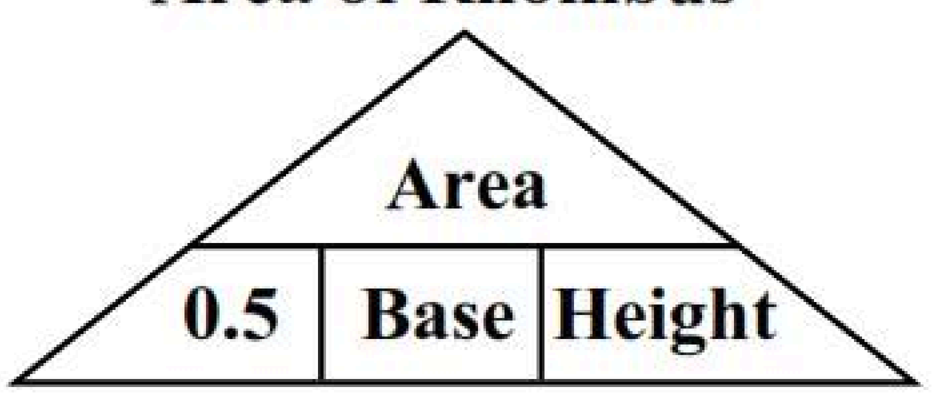


Rhombus : Area = Base (L) × H = 0.5 × D1 × D2

: Perimeter = length × 4

: L = $\sqrt{Area}$ **= Perimeter ÷ 4**

Area of Rhombus

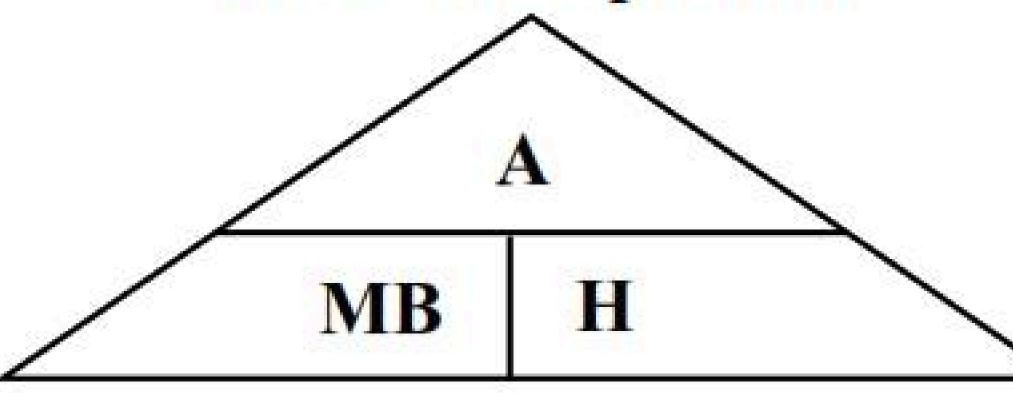


Trapezium: Middle Base = $\frac{B1 + B2}{2}$

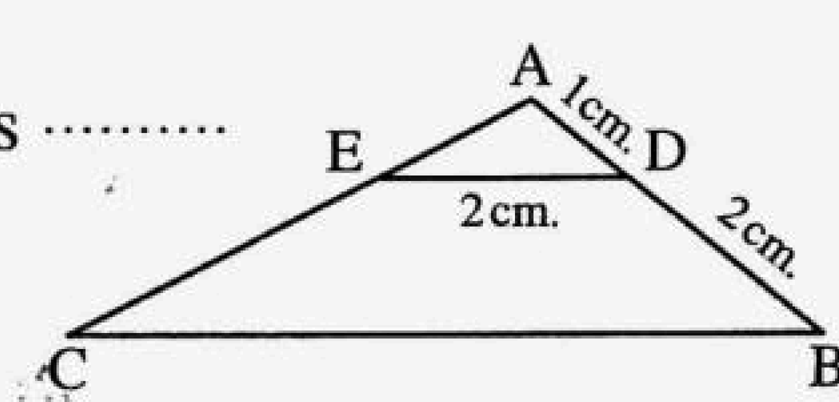
: A = MB × H = or $\frac{B1 + B2}{2} \times H$

B1 = 2 × MB – B2

Area of Trapezium



[A] : Choose The Correct Answer : -

No.	Question
24	In the two similar polygons their corresponding angles are .... in measure (a) equal (b) difference (c) proportional (d) alternatives
25	The ratio between the lengths of two corresponding sides of two similar polygons is 3 : 5 , then the ratio between their perimeters is .... (a) 5 : 2 (b) 5 : 3 (c) 1 : 2 (d) 3 : 5
26	The ratio between the lengths of corresponding sides of two similar triangles is 3 : 5 and if the perimeter of the greater triangle is 60 cm. then the perimeter of the smaller is .... cm. (a) 24 (b) 36 (c) 40 (d) 100
27	If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of smaller polygon is 15 cm. , then the perimeter of the greater polygon is .... cm. (a) 30 (b) 45 (c) 60 (d) 75
28	If the ratio of enlargement between two triangles equals 1 , then the two triangles are .... (a) congruent. (b) different. (c) right-angle. (d) coincide.
29	If $\Delta ABC \sim \Delta DEF$ and $m(\angle B) + m(\angle C) = 70°$, then $m(\angle D) =$ .... (a) 70° (b) 90° (c) 110° (d) 180°
30	**In the opposite figure :** If $\Delta ADE \sim \Delta ABC$, then the length of $\overline{BC}$ in cm. equals .... (a) 3 (b) 4 (c) 6 (d) 8 

31

In the opposite figure :

If $\Delta ADE \sim \Delta ABC$, then BC = ·········· cm.

(a) 6 (b) 8

(c) 9 (d) 12

32

If the projection of a line segment on a straight line is a point , then the line segment ·········· the straight line.

(a) // (b) $\perp$ (c) $\equiv$ (d) $\supset$

33

ABCD is square the projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is ··········

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{CD}$ (d) $\overline{AD}$

34

If : $\overline{AB}$ // $\overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ ·········· the length of $\overline{AB}$

(a) < (b) > (c) = (d) $\leq$

35

Δ ABC is right-angled at B , $\overline{BD} \perp \overline{AC}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is ··········

(a) $\{A\}$ (b) $\{B\}$ (c) $\{C\}$ (d) $\{D\}$

36

If : $\overline{AB} \perp \overleftrightarrow{CD}$, then the length of the projection $\overline{AB}$ on $\overleftrightarrow{CD}$ equals ··········

(a) 1 (b) 0 (c) CD (d) AB

37

The projection of point (3 , 5) on the x - axis is ··········

(a) (5 , 0) (b) (0 , 5) (c) (0 , 3) (d) (3 , 0)

38

The triangle whose sides have lengths 3 cm. , 5 cm. , 4 cm. is ··········

(a) right-angled. (b) acute-angled. (c) obtuse-angled. (d) otherwise.

39

In the triangle whose side lengths are 5 cm. , 4 cm. , 7 cm. , the greatest angle in measure is ·········· angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

40	The triangle whose side lengths 6 cm. , 7 cm. and 3 cm. is .... triangle. (a) equilateral. (b) obtuse-angled. (c) right-angled (d) acute-angled
41	ABC is an acute-angled triangle in which AB = 6 cm. , BC = 8 cm. , then AC = ... cm. (a) 2 (b) 6 (c) 10 (d) 14
42	In Δ ABC if $(AB)^2 = (AC)^2 + (BC)^2$, then ($\angle$ C) is .... (a) right. (b) acute. (c) obtuse. (d) straight.
43	Δ ABC in which $(AC)^2 = (BC)^2 - (AB)^2$, Then the angle A is .... angle. (a) acute (b) right (c) obtuse (d) straight
44	In Δ ABC if $(AB)^2 > (AC)^2 + (BC)^2$, then ($\angle$ C) is .... angle. (a) right (b) acute (c) obtuse (d) reflex
45	In Δ ABC if $(AC)^2 + (AB)^2 < (BC)^2$, then type of $\angle$ A is .... (a) an obtuse. (b) straight. (c) an acute. (d) right.
46	ABCD is a parallelogram , E $\in \overline{BC}$, then the area of ▱ ABCD .... The area of Δ EAD (a) the same (b) half (c) twice (d) third
47	A rectangle whose perimeter is 28 cm. and its length is 8 cm. , then the length of its diagonal = ... cm. (a) 56 (b) 48 (c) 10 (d) 216
48	The ratio between area of parallelogram and area of triangle if they have a common base and including between two parallel lines equals .... (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 2 : 3
49	The median of a triangle divides its surface into two .... (a) congruent triangles. (b) triangular surfaces equal in area. (c) isosceles triangles. (d) right-angled triangles.

50	The median of a triangle divides it surface into two triangles that are ……… (a) equal in area (b) similar (c) different in area (d) congruent
51	**In the opposite figure :** m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$, CD = 2 cm. , BD = 6 cm. , then AC = ……… cm. (a) 2 (b) 4 (c) 3 (d) 12 A B 6cm. D 2cm. C
52	**In the opposite figure :** If : YL = 4 cm. and LZ = 5 cm. , then XY = ……… cm. (a) $3\sqrt{5}$ (b) 20 (c) 9 (d) 6 X Z 5cm. L 4cm. Y

[B] : Complete the Following : -

1	If the point A ∈ the line L , then the projection of the point A on the line L is ………
2	If the point lies on the straight line , then its projection on it is ………
3	If : $\overline{AB}$ // $\overleftrightarrow{XY}$, then the length of projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ ……… AB
4	In the square ABCD the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ………
5	In the square ABCD the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ………
6	If Δ ABC is a right-angled at ∠ B , then the projection of $\overline{BC}$ on $\overline{AB}$ is ………
7	XYZ is triangle in which $(XZ)^2 = (XY)^2 + (ZY)^2$, then m (∠ ………) = 90°
8	ABC is a triangle if $(AB)^2 = (BC)^2 + (AC)^2$, then m (∠ ………) = 90°

9	XYZ is triangle in which $(XY)^2 = (YZ)^2 - (XZ)^2$, then angle .... is right.
10	In the triangle Δ XYZ if $(XY)^2 > (XZ)^2 + (YZ)^2$, then angle Y is ....
11	In Δ ABC if $(AC)^2 > (AB)^2 + (BC)^2$, then Δ ABC is an .... triangle.
12	In Δ ABC if $(AC)^2 + (AB)^2 < (BC)^2$, then angle A is ....
13	The triangle of side lengths 3 cm. , 4 cm. , 5 cm. is .... angled-triangle.
14	In a triangle if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides , then the angle opposite to this side is ....

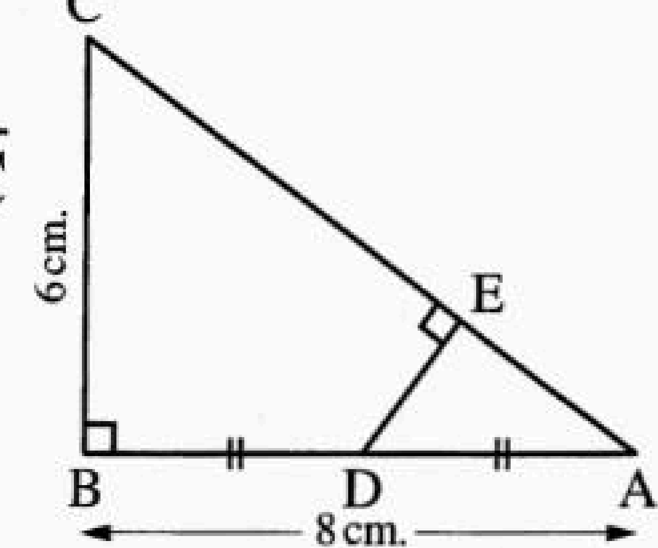
[C] : Essay Problems : -

1

In the opposite figure :

Δ ABC is a right - angled at B , D is midpoint of $\overline{AB}$, $\overline{DE} \perp \overline{AC}$, AB = 8 cm. , BC = 6 cm.

Prove that : Δ AED ~ Δ ABC , then find the length of $\overline{DE}$



2

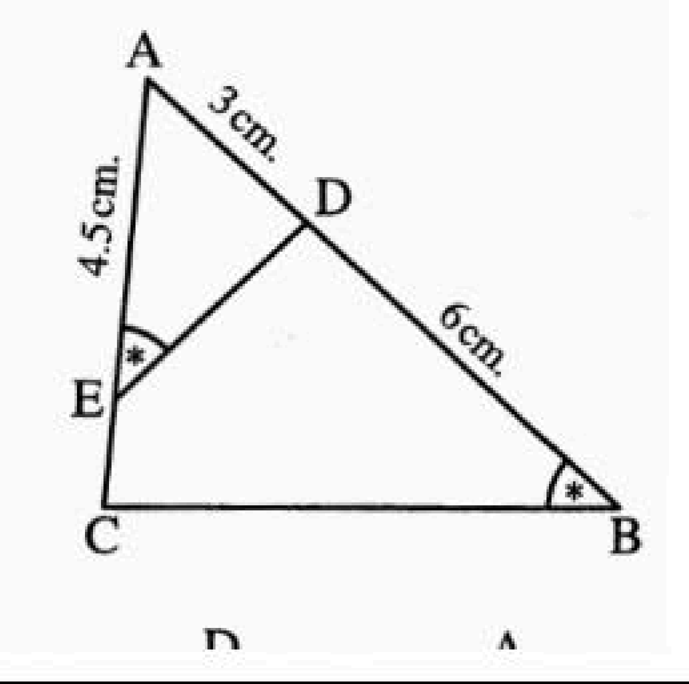
In the opposite figure :

m (∠ AED) = m (∠ B) , AD = 3 cm.

, AE = 4.5 cm. and BD = 6 cm.

① **Prove that :** Δ ADE ~ Δ ACB

② **Find the length of :** $\overline{CE}$



3

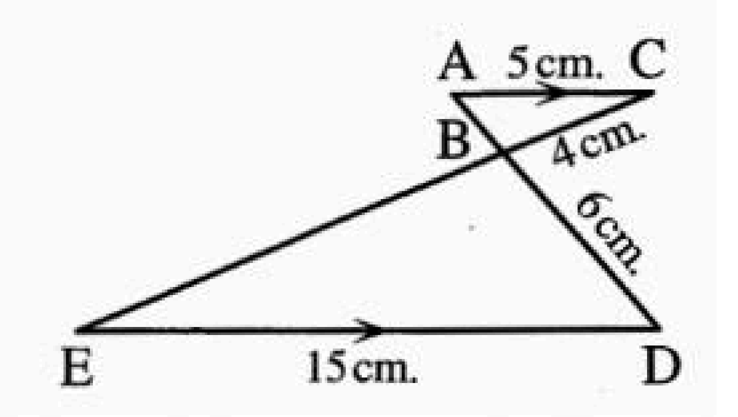
In the opposite figure :

$\overline{AC}$ // $\overline{ED}$, AC = 5 cm.

, BC = 4 cm. , BD = 6 cm. , ED = 15 cm.

① **Prove that :** Δ ABC ~ Δ DBE

② **Find the length of each of :** $\overline{AB}$ and $\overline{BE}$



4	**In the opposite figure :** $\overline{DE} \parallel \overline{BC}$, AD = 6 cm. , BD = 9 cm. and DE = 4 cm. ① **Prove that :** Δ ADE ~ Δ ABC ② **Find the length of :** $\overline{BC}$
5	**In the opposite figure :** Δ ABC ~ Δ ADE , AE = 6 cm. , AD = 7 cm. , BE = 8 cm. **Find the length of :** $\overline{DC}$
6	**In the opposite figure :** ABC is a triangle in which : BD = 2 cm. , CD = 8 cm. , AD = 4 cm. , $\overline{AD} \perp \overline{BC}$ **Prove that :** m ($\angle$ BAC) = 90°
7	**In the opposite figure :** ABC is a triangle , $\overline{AD} \perp \overline{BC}$ If AD = 24 cm. , AB = 26 cm. and AC = 30 cm. **Find :** BC , then calculate area of Δ ABC
8	**In the opposite figure :** ABC is a right-angled triangle at A , $\overline{AD} \perp \overline{BC}$, BD = 9 cm. , CD = 16 cm. **Find the length of each of :** $\overline{AB}$, $\overline{AC}$, $\overline{AD}$
9	**In the opposite figure :** ABCD is a quadrilateral in which : AB = 8 cm. , BC = 9 cm. , CD = 12 cm. and AD = 17 cm. , m ($\angle$ ABD) = 90° ① **Find the length of :** $\overline{BD}$ ② **Prove that :** $\angle$ C is a right angle

10	The sides lengths of one of two similar triangles are 3 cm. , 4 cm. , 5 cm. and the perimeter of the other triangle is 36 cm. find the side lengths of the other triangle.
11	Determine the type of angle C in Δ ABC in which : AB = 7 cm. , BC = 3 cm. and AC = 5 cm.



Prep (2) : Second Term (2016)

Geometry : Final Revision Solutions

[A] Choose Problems Answers

Sn.	Answer	Sn.	Answer
1	Equal	16	An obtuse
2	3 : 5	17	An obtuse
3	36	18	6
4	45	19	Right
5	Congruent	20	Right
6	110	21	Obtuse
7	6	22	An obtuse
8	9	23	Twice
9	⊥	24	10
10	BC	25	2 : 1
11	=	26	Answer (b)
12	{ D }	27	Equal in area
13	0	28	4
14	(3 , 0)	29	6
15	Right angle		

[B] Complete Problems Answers

Sn.	Answer	Sn.	Answer
1	A	5	DC
2	Itself	6	BA
3	=	7	Y
4	BC	8	C
9	Y	12	Obtuse
10	Acute	13	Right
11	Obtuse	14	right

Essay Problems

Sn	Answer
1	In Δ ABC : ∵ m (∠ B) = 90° ∴ $(AC)^2 = (AB)^2 + (BC)^2 = 64 + 36 = 100$ ∴ AC = 10 cm. *In ΔΔ AED , ABC : ∵ m (∠ AED) = m (∠ B) = 90°* , ∠ A is a common angle ∴ m (∠ ADE) = m (∠ C) ∴ Δ AED ∼ Δ ABC (First req.) ∴ $\frac{AE}{AB} = \frac{ED}{BC} = \frac{AD}{AC}$ ∴ $\frac{ED}{6} = \frac{4}{10}$ ∴ $ED = \frac{6 \times 4}{10} = 2.4$ cm. (Second req.)
2	In ΔΔ ADE , ACB : ∵ m (∠ AED) = m (∠ ABC) , ∠ A is a common angle ∴ m (∠ ADE) = m (∠ C) ∴ Δ ADE ∼ Δ ACB (First req.) ∴ $\frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$ ∴ $\frac{3}{AC} = \frac{4.5}{9}$ ∴ $AC = \frac{3 \times 9}{4.5} = 6$ cm. ∴ CE = 6 – 4.5 = 1.5 cm. (Second req.)
3	In ΔΔ ABC , DBE : ∵ $\overline{AC}$ // $\overline{DE}$, $\overleftrightarrow{CE}$ is a transversal ∴ m (∠ C) = m (∠ E) (Alternate angles) (1) , ∵ $\overline{AC}$ // $\overline{DE}$, $\overleftrightarrow{AD}$ is a transversal ∴ m (∠ A) = m (∠ D) (Alternate angles) (2) , ∵ m (∠ ABC) = m (∠ DBE) (V.O.A) (3) From (1) , (2) and (3) : ∴ Δ ABC ∼ Δ DBE (First req.) ∴ $\frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$ ∴ $\frac{AB}{6} = \frac{4}{BE} = \frac{5}{15}$ ∴ $AB = \frac{6 \times 5}{15} = 2$ cm. , $BE = \frac{4 \times 15}{5} = 12$ cm. (Second req.)

4

In $\Delta\Delta$ ADE , ABC :

$\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle ADE) = m(\angle B)$ (Corresponding angles) (1)

, $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ *is a transversal*

$\therefore m(\angle AED) = m(\angle C)$ (Corresponding angles) (2)

From (1) and (2) : , $\because \angle A$ is a common angle

$\therefore \Delta ADE \sim \Delta ABC$ (First req.)

$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{6}{15} = \frac{4}{BC}$

$\therefore BC = \frac{15 \times 4}{6} = 10$ cm. (Second req.)

5

$\because \Delta ABC \sim \Delta ADE$

$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$ $\therefore \frac{14}{7} = \frac{AC}{6}$

$\therefore AC = \frac{6 \times 14}{7} = 12$ cm.

$\therefore DC = 12 - 7 = 5$ cm. (The req.)

6

$\because \Delta ABD$ is right-angled at D

$\therefore (AB)^2 = (AD)^2 + (DB)^2 = (4)^2 + (2)^2 = 20$

, $\because \Delta ADC$ is right-angled at D

$\therefore (AC)^2 = (AD)^2 + (DC)^2 = (4)^2 + (8)^2 = 80$

In ΔABC : $\because (AB)^2 + (AC)^2 = 20 + 80 = 100$

, $(BC)^2 = 100$

$\therefore m(\angle BAC) = 90°$ (Q.E.D.)

7

In ΔABD : $\because m(\angle ADB) = 90°$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

$\therefore BD = 10$ cm.

, In ΔACD : $\because m(\angle ADC) = 90°$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore CD = 18$ cm.

$\therefore BC = CD + DB = 18 + 10 = 28$ cm. (First req.)

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 28 \times 24 = 336 \text{ cm}^2$.

(Second req.)

8

In ΔABC : $\because m(\angle BAC) = 90°$, $\overline{AD} \perp \overline{BC}$

$\therefore (AB)^2 = BD \times BC = 9 \times 25 = 225$

$\therefore AB = 15$ cm.

, $(AC)^2 = CD \times CB = 16 \times 25 = 400$

$\therefore AC = 20$ cm.

, $(AD)^2 = DB \times DC = 9 \times 16 = 144$

$\therefore AD = 12$ cm. (The req.)

9

In ΔABD : $\because m(\angle ABD) = 90°$

$\therefore (BD)^2 = (AD)^2 - (AB)^2 = (17)^2 - (8)^2 = 225$

$\therefore BD = 15$ cm. (First req.)

In ΔBCD : $\because (BD)^2 = 225$

, $(BC)^2 + (CD)^2 = (9)^2 + (12)^2 = 225$

$\therefore (BD)^2 = (BC)^2 + (CD)^2$

$\therefore m(\angle C) = 90°$ (Second req.)

10

Let the two triangles be ΔABC , ΔXYZ

, $\because \Delta ABC \sim \Delta XYZ$

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta XYZ}$

$\therefore \frac{3}{XY} = \frac{4}{YZ} = \frac{5}{XZ} = \frac{12}{36}$ $\therefore XY = \frac{3 \times 36}{12} = 9$ cm.

, $YZ = \frac{4 \times 36}{12} = 12$ cm. , $XZ = \frac{5 \times 36}{12} = 15$ cm.

11

$\because (AB)^2 = 7^2 = 49$, $(BC)^2 + (AC)^2 = 3^2 + 5^2 = 34$

$\therefore (AB)^2 > (BC)^2 + (AC)^2$ $\therefore \angle C$ is obtuse

Prep (2) : Second Term (2016)

Geometry : Final Revision Solutions

[A] Choose Problems Answers

Sn.	Answer	Sn.	Answer
1	Equal	16	An obtuse
2	3 : 5	17	An obtuse
3	36	18	6
4	45	19	Right
5	Congruent	20	Right
6	110	21	Obtuse
7	6	22	An obtuse
8	9	23	Twice
9	⊥	24	10
10	BC	25	2 : 1
11	=	26	Answer (b)
12	{ D }	27	Equal in area
13	0	28	4
14	(3 , 0)	29	6
15	Right angle		

[B] Complete Problems Answers

Sn.	Answer	Sn.	Answer
1	A	5	DC
2	Itself	6	BA
3	=	7	Y
4	BC	8	C

9	Y	12	Obtuse
10	Acute	13	Right
11	Obtuse	14	right

Essay Problems

Sn 1 – Answer

In Δ ABC : ∵ m (∠ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 64 + 36 = 100$

∴ AC = 10 cm.

In ΔΔ AED , ABC : ∵ m (∠ AED) = m (∠ B) = 90°

, ∠ A is a common angle

∴ m (∠ ADE) = m (∠ C)

∴ Δ AED ~ Δ ABC (First req.)

$\therefore \frac{AE}{AB} = \frac{ED}{BC} = \frac{AD}{AC}$ $\therefore \frac{ED}{6} = \frac{4}{10}$

$\therefore ED = \frac{6 \times 4}{10} = 2.4$ cm. (Second req.)

Sn 2 – Answer

In ΔΔ ADE , ACB :

∵ m (∠ AED) = m (∠ ABC)

, ∠ A is a common angle

∴ m (∠ ADE) = m (∠ C)

∴ Δ ADE ~ Δ ACB (First req.)

$\therefore \frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$ $\therefore \frac{3}{AC} = \frac{4.5}{9}$

$\therefore AC = \frac{3 \times 9}{4.5} = 6$ cm.

∴ CE = 6 – 4.5 = 1.5 cm. (Second req.)

Sn 3 – Answer

In ΔΔ ABC , DBE :

∵ $\overline{AC} \parallel \overline{DE}$, $\overleftrightarrow{CE}$ is a transversal

∴ m (∠ C) = m (∠ E) (Alternate angles) (1)

, ∵ $\overline{AC} \parallel \overline{DE}$, $\overleftrightarrow{AD}$ is a transversal

∴ m (∠ A) = m (∠ D) (Alternate angles) (2)

, ∵ m (∠ ABC) = m (∠ DBE) (V.O.A) (3)

From (1) , (2) and (3) :

∴ Δ ABC ~ Δ DBE (First req.)

$\therefore \frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$ $\therefore \frac{AB}{6} = \frac{4}{BE} = \frac{5}{15}$

$\therefore AB = \frac{6 \times 5}{15} = 2$ cm.

$, BE = \frac{4 \times 15}{5} = 12$ cm. (Second req.)

4	In $\Delta\Delta$ ADE , ABC : $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal $\therefore m(\angle ADE) = m(\angle B)$ (Corresponding angles) (1) , $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal $\therefore m(\angle AED) = m(\angle C)$ (Corresponding angles) (2) From (1) and (2) : , $\because \angle A$ is a common angle $\therefore \Delta ADE \sim \Delta ABC$ (First req.) $\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{6}{15} = \frac{4}{BC}$ $\therefore BC = \frac{15 \times 4}{6} = 10$ cm. (Second req.)
5	$\because \Delta ABC \sim \Delta ADE$ $\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$ $\therefore \frac{14}{7} = \frac{AC}{6}$ $\therefore AC = \frac{6 \times 14}{7} = 12$ cm. $\therefore DC = 12 - 7 = 5$ cm. (The req.)
6	$\because \Delta ABD$ is right-angled at D $\therefore (AB)^2 = (AD)^2 + (DB)^2 = (4)^2 + (2)^2 = 20$, $\because \Delta ADC$ is right-angled at D $\therefore (AC)^2 = (AD)^2 + (DC)^2 = (4)^2 + (8)^2 = 80$ In ΔABC : $\because (AB)^2 + (AC)^2 = 20 + 80 = 100$, $(BC)^2 = 100$ $\therefore m(\angle BAC) = 90°$ (Q.E.D.)
7	In ΔABD : $\because m(\angle ADB) = 90°$ $\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$ $\therefore BD = 10$ cm. , In ΔACD : $\because m(\angle ADC) = 90°$ $\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$ $\therefore CD = 18$ cm. $\therefore BC = CD + DB = 18 + 10 = 28$ cm. (First req.) $\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$ $= \frac{1}{2} \times 28 \times 24 = 336$ cm^2. (Second req.)
8	In ΔABC : $\because m(\angle BAC) = 90°$, $\overline{AD} \perp \overline{BC}$ $\therefore (AB)^2 = BD \times BC = 9 \times 25 = 225$ $\therefore AB = 15$ cm. , $(AC)^2 = CD \times CB = 16 \times 25 = 400$ $\therefore AC = 20$ cm. , $(AD)^2 = DB \times DC = 9 \times 16 = 144$ $\therefore AD = 12$ cm. (The req.)
9	In ΔABD : $\because m(\angle ABD) = 90°$ $\therefore (BD)^2 = (AD)^2 - (AB)^2 = (17)^2 - (8)^2 = 225$ $\therefore BD = 15$ cm. (First req.) In ΔBCD : $\because (BD)^2 = 225$, $(BC)^2 + (CD)^2 = (9)^2 + (12)^2 = 225$ $\therefore (BD)^2 = (BC)^2 + (CD)^2$ $\therefore m(\angle C) = 90°$ (Second req.)
10	Let the two triangles be ΔABC , ΔXYZ , $\because \Delta ABC \sim \Delta XYZ$ $\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta XYZ}$ $\therefore \frac{3}{XY} = \frac{4}{YZ} = \frac{5}{XZ} = \frac{12}{36}$ $\therefore XY = \frac{3 \times 36}{12} = 9$ cm. , $YZ = \frac{4 \times 36}{12} = 12$ cm. , $XZ = \frac{5 \times 36}{12} = 15$ cm.
11	$\because (AB)^2 = 7^2 = 49$, $(BC)^2 + (AC)^2 = 3^2 + 5^2 = 34$ $\therefore (AB)^2 > (BC)^2 + (AC)^2$ $\therefore \angle C$ is obtuse

RULES OF GEOMETRY

Equality of the areas of two parallelograms

The altitude of the parallelogram

- The altitude is a line segment , or the length of a line segment , giving the height of a polygon.
- **In the opposite figure :**

  ABCD is a parallelogram ,
  $F \in \overline{CB}$ such that $\overline{DF} \perp \overline{CB}$,
  $E \in \overline{AB}$ such that $\overline{DE} \perp \overline{AB}$,
  then :
  - The length of $\overline{DF}$ is the altitude (height) corresponding to the base $\overline{BC}$
  - The lenght of $\overline{DE}$ is the altitude (height) corresponding to the base $\overline{AB}$

Notice that :

We can consider any side in the parallelogram ABCD is a base , then :

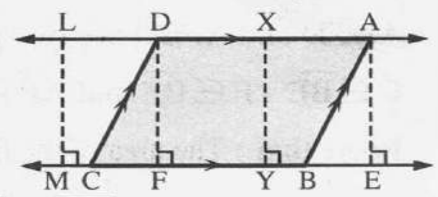
- The altitude corresponding to the base $\overline{BC}$ is the same altitude corresponding to the base $\overline{AD}$
- The altitude corresponding to the base $\overline{AB}$ is the same altitude corresponding to the base $\overline{CD}$

Remark

The perpendicular distance between any two parallel straight lines is constant , then :

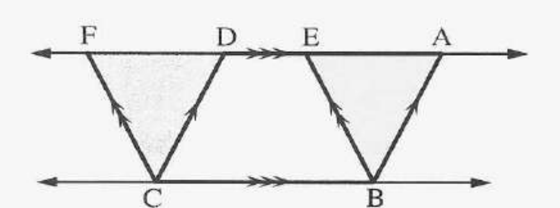
In the opposite figure :

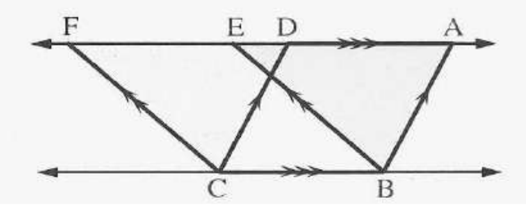
AE = XY = DF = LM and each of them is considered an altitude of the parallelogram ABCD corresponding to $\overline{BC}$ or $\overline{AD}$



Theorem 1

Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are equal in area.



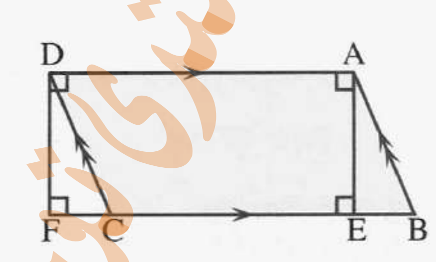


Corollary 1

The parallelogram and the rectangle with common base and between two parallel straight lines are equal in area.

- **In the opposite figure :**

The area of the parallelogram ABCD
= the area of the rectangle AEFD
(They have a common base $\overline{AD}$
and they are between the two parallel straight lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$)



Try to prove this corollary in the same way of proving the previous theorem.

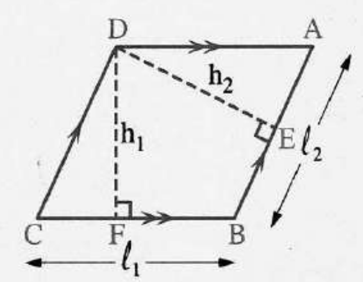
Corollary 2

The area of the parallelogram = the length of the base × its corresponding height.

Remark

In the opposite figure :

If ABCD is a parallelogram , DF is the corresponding height of the base $\overline{BC}$ and DE is the corresponding height of the base $\overline{AB}$, then : The area of the parallelogram ABCD = BC × DF = AB × DE



i.e. $\ell_1 \times h_1 = \ell_2 \times h_2$

Corollary 3

The parallelograms with bases equal in length and lying on a straight line , while the opposite sides to these bases are on another straight line , are equal in area.

Corollary 4

Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.

Remark

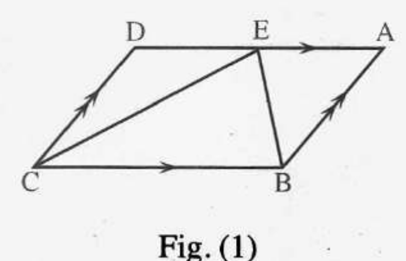


Fig. (1)

Fig. (2)

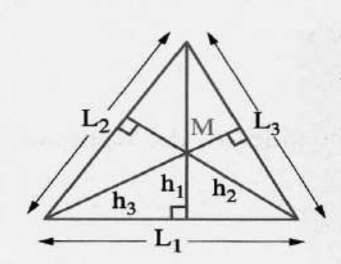
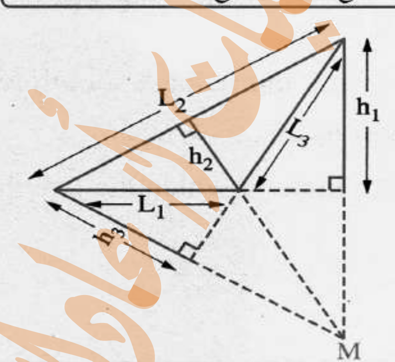
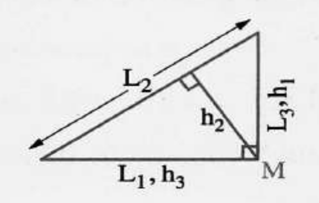
In each of the previous figures , the area of Δ BCE = $\frac{1}{2}$ of the area of ▱ ABCD

Corollary 5

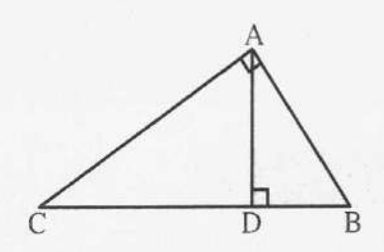
Area of the triangle = $\frac{1}{2}$ of the length of the base × its corresponding height

Remark

Any triangle has three sides , each of them is called a base and each base has a corresponding altitude , the straight lines carrying these altitudes intersect at one point as shown in the following figures :

The acute-angled triangle	The obtuse-angled triangle	The right-angled triangle
		
They intersect at a point inside the triangle.	They intersect at a point outside the triangle.	They intersect at the vertex of the right angle.

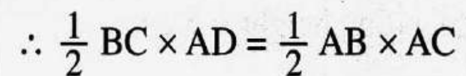
Remark



If Δ ABC is right-angled at A and D $\in \overline{BC}$

such that : $\overline{AD} \perp \overline{BC}$

Then the area of Δ ABC = $\frac{1}{2}$ BC × AD = $\frac{1}{2}$ AB × AC



$\therefore \frac{1}{2} BC \times AD = \frac{1}{2} AB \times AC$

$\therefore BC \times AD = AB \times AC$

Equality of the areas of two triangles

- **We knew in the previous lesson that :**

  The area of the triangle = $\frac{1}{2}$ of the base length × its corresponding height.

According to this , we can say :

If the lengths of the two bases of two triangles are equal and their corresponding heights are equal , then the areas of the two triangles are equal.

In this lesson , we shall study some different cases of the equality of two areas of two triangles.

Theorem 2

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.

Important corollaries

Corollary 1

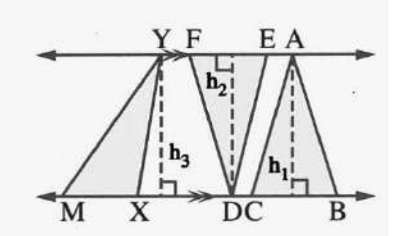
Triangles of bases equal in length and lying between two parallel straight lines are equal in area.

In the opposite figure :

If $\overleftrightarrow{AE} \parallel \overleftrightarrow{BC}$ and BC = EF = XM ,

then the area of Δ ABC = the area of Δ DEF

= the area of Δ YXM



Notice that: $h_1 = h_2 = h_3$

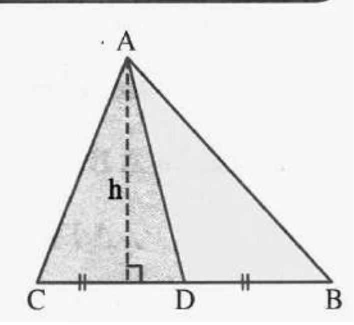
Corollary 2

The median of a triangle divides its surface into two triangular surfaces equal in area.

In the opposite figure :

If $\overline{AD}$ is a median in Δ ABC ,

then the area of Δ ABD = the area of Δ ADC



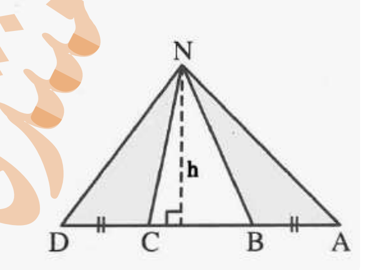
Notice that: The two triangles have the same height h and BD = DC

Corollary 3

Triangles with congruent bases on one straight line and have a common vertex are equal in areas.

In the opposite figure :

The area of Δ NAB = the area of Δ NCD



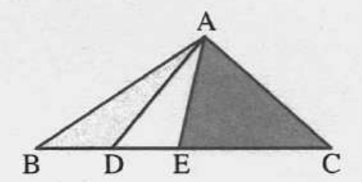
Notice that :

The two triangles have the same height (h) and AB = CD

Remark

In the opposite figure :

If BD = $\frac{1}{2}$ EC , then the area of Δ ABD = $\frac{1}{2}$ the area of Δ AEC



Theorem 3

If two triangles are equal in area and drawn on the same base and on one side of it , then their vertices lie on a straight line parallel to this base.

Remark

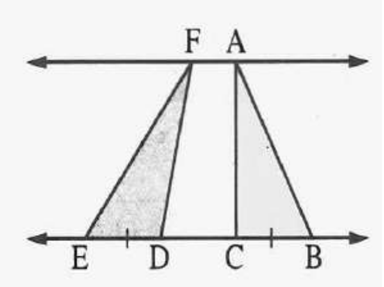
If two triangles have the same area and they are included between two straight lines and their bases on these two straight lines are equal in length , then the two straight lines are parallel.

1 In the opposite figure :

If B , C , D and E are on a straight line ,

BC = DE , the area of Δ ABC = the area of Δ FDE

, then $\overleftrightarrow{AF} \parallel \overleftrightarrow{BE}$

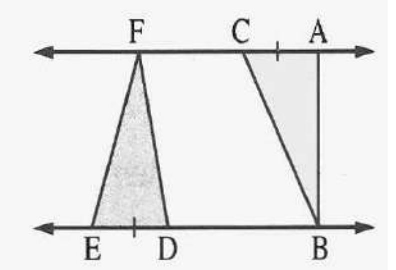


2 In the opposite figure :

If C ∈ $\overline{AF}$, D ∈ $\overline{BE}$, AC = DE ,

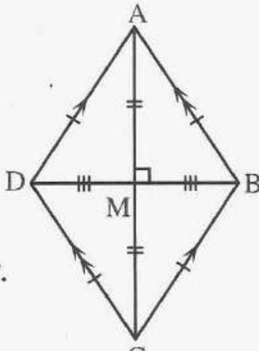
the area of Δ ABC = the area of Δ FDE

, then $\overleftrightarrow{AF} \parallel \overleftrightarrow{BE}$



Areas of some geometric figures

(1) Rhombus



* The rhombus is a parallelogram whose sides are equal in length.

i.e. • $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$ • AB = BC = CD = DA

* The two diagonals of the rhombus are perpendicular and bisect each other.

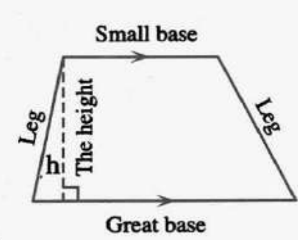
i.e. • $\overline{AC} \perp \overline{BD}$ • AM = CM , BM = DM

Remark

∵ The square is a rhombus with two equal diagonals in length.

∴ The area of the square = $\frac{1}{2}$ of the square of the length of its diagonal.

(2) The trapezium (The trapezoid)



It is a quadrilateral in which two sides are parallel.

- The two parallel sides are called the bases of the trapezium.
- The other two sides are called the two legs of the trapezium.
- The trapezium has one height only which is the perpendicular distance between its two bases (h)

The isosceles trapezium

If the two legs of the trapezium are equal in length, then it is called an isosceles trapezium.
The following are the properties of the isosceles trapezium :

(1) The two base angles of the isosceles trapezium are equal in measure.

In the opposite figure :

If $\overline{AD} \parallel \overline{BC}$ and AB = DC,

then :

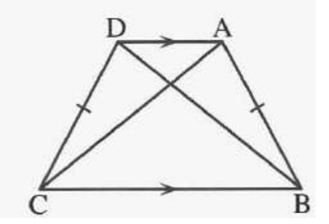
m (∠ B) = m (∠ C) and m (∠ A) = m (∠ D)

(2) The two diagonals of the isosceles trapezium are equal in length.

In the opposite figure :

If $\overline{AD} \parallel \overline{BC}$ and AB = DC,

then AC = BD

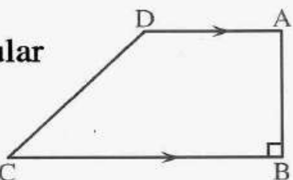


(3) The isosceles trapezium has only one axis of symmetry which is the perpendicular bisector of its bases.

Notice that:

The axis of symmetry of the isosceles trapezium passes through the point of intersection of its two diagonals.

The right trapezium



- A right trapezium is a trapezium whose one of its legs is perpendicular to its two parallel bases.
- In this case , the length of this perpendicular leg is the height of the trapezium.

The middle base of the trapezium

The length of the middle base = $\frac{1}{2}$ the sum of the two lengths of the two parallel bases.

The figure		The perimeter	The area
The triangle		The sum of the lengths of its three sides	$\frac{1}{2}$ of the base length × height $= \frac{1}{2}\ \ell \times h$
The parallelogram		The sum of lengths of two adjacent sides × 2 $= 2(\ell_1 + \ell_2)$	The base length × height $= \ell_1 \times h_1 = \ell_2 \times h_2$
The rectangle		2 (Length + Width) $= 2(\ell + w)$	Length × Width $= \ell \times w$
The square		Side length × 4 = 4 ℓ	Square of side length = ℓ^2 or $\frac{1}{2}$ of the square of its diagonal length = $\frac{1}{2}\ r^2$
The rhombus		Side length × 4 = 4 ℓ	Side length × height = $\ell \times h$ or $\frac{1}{2}$ the product of the lengths of the two diagonals = $\frac{1}{2}\ r_1 \times r_2$
The trapezium		The sum of lengths of its sides	$\frac{1}{2}$ the sum of lengths of the two parallel bases × height $= \frac{1}{2}(\ell_1 + \ell_2) \times h$ or the length of the middle base × height $= \ell \times h$

Similarity

Similarity of two polygons

Definition

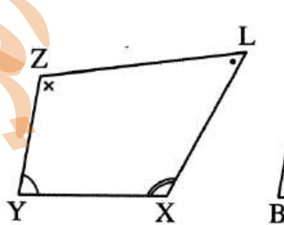
It is said that the two polygons P_1 and P_2 (of the same number of sides) are similar if the following two conditions are verified together :

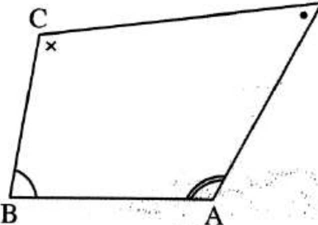
1. Their corresponding angles are equal in measure.
2. Their corresponding side lengths are proportional.

In this case , we write the polygon P_1 ~ the polygon P_2

That means the polygon P_1 is similar to the polygon P_2

In the opposite figure :





1. $m(\angle A) = m(\angle X)$
   , $m(\angle B) = m(\angle Y)$
   , $m(\angle C) = m(\angle Z)$
   , $m(\angle D) = m(\angle L)$

i.e.

The measures of the corresponding angles are equal.

2. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$ = constant.

i.e.

The lengths of the corresponding sides are proportional

, then from 1 and 2 , we deduce that : the polygon ABCD ~ the polygon XYZL

Remark (1)

In the two similar polygons P_1 and P_2 , the constant ratio among the lengths of the corresponding sides of P_1 and P_2 is called the ratio of enlargement or the drawing scale.

If the constant ratio is :

- Greater than 1 , then the polygon P_1 is an enlargement to the polygon P_2
- Less than 1 , then the polygon P_1 is a minimizing of the polygon P_2
- Equal to 1 , then the polygon P_1 is congruent to the polygon P_2

Remark (2)

In order that two polygons are similar , the two conditions should be verified together and verifying one of them only is not enough to be similar.

For example:

- The square and the rectangle are not similar polygons although the measures of their corresponding angles are equal (each of them is a right angle) but their corresponding side lengths are not proportional.
- So the square and the rhombus are not similar polygons although their corresponding side lengths are proportional but the measures of their corresponding angles are not equal.
  In the square , each angle is a right angle but in the rhombus that doesn't exist.

Remark (3)

The congruent polygons are similar but it is not necessary that the similar polygons are congruent.

Remark (4)

All regular polygons of the same number of sides are similar.

For example: All squares are similar.

Remark (5)

If each of two polygons is similar to a third polygon , then they are similar.

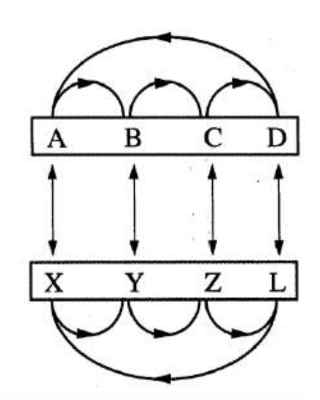
Remark (6)

The order of corresponding vertices should be kept in giving names of similar polygons that to help us finding the proportional sides lengths and the equal angles in measures.

For example:

If we write P_1 (ABCD) is similar to P_2 (XYZL) ,

then we deduce directly that :



1 $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$

2 $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$,
$m(\angle C) = m(\angle Z)$, $m(\angle D) = m(\angle L)$

i.e.

The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides.

Similarity of two triangles

- We knew that for two polygons in order to be similar, two conditions should be verified together, one of them is not enough to say that the two polygons are similar. But in triangles, the following fact shows that the two conditions will be verified together if one of them is verified.

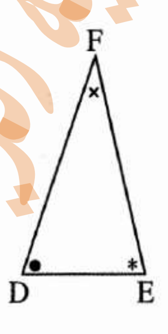
A geometric fact :

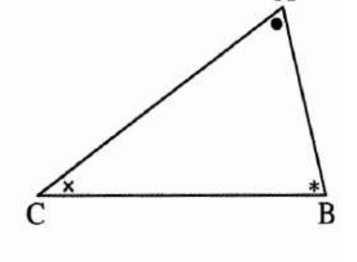
The two triangles are similar if one of the two following conditions is verified :

1. The measures of their corresponding angles are equal.
2. The lengths of their corresponding sides are proportional.

For example:

- **In the opposite figure :**





$\Delta ABC \sim \Delta DEF$ because :

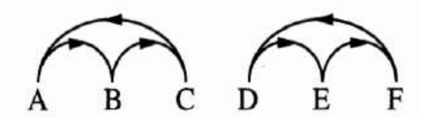
$m(\angle A) = m(\angle D)$,

$m(\angle B) = m(\angle E)$,

$m(\angle C) = m(\angle F)$

As a result for their similarity , we find that :

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$



Remarks

1. The two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other.
2. The two equilateral triangles are similar.
3. The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.

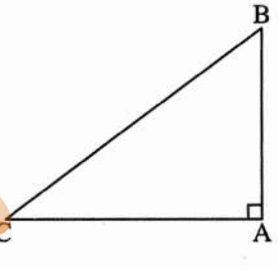
Converse of Pythagoras theorem

We studied Pythagoras' theorem last year.

In the following , we will remind you of what you have studied.

If ABC is a right-angled triangle at A, then $(BC)^2 = (AB)^2 + (AC)^2$

Now we shall study the converse of Pythagoras' theorem.

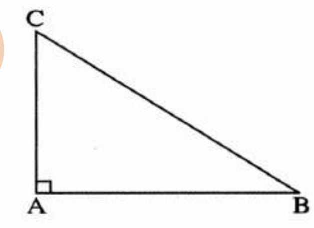


The converse of Pythagoras' theorem

In Δ ABC ,

if $(AB)^2 + (AC)^2 = (BC)^2$,

then $m(\angle A) = 90°$



We can state this theorem as follows :

In a triangle, if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

Corollary

In Δ ABC , if $\overline{AC}$ is the longest side and if $(AC)^2 \neq (AB)^2 + (BC)^2$, then $m(\angle B) \neq 90°$ and the triangle is not right-angled.

Projections

① The projection of a point on a straight line

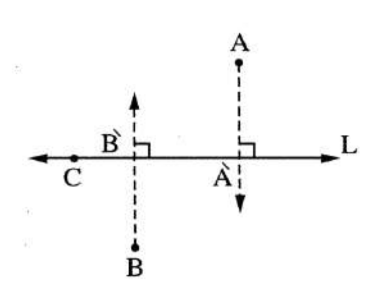
- **In the opposite figure :**

L is a straight line, the two points

A and B are not belonging to the straight line L

Draw from A the ray $\overrightarrow{AA'} \perp L$ to cut L at A`

Then draw from B the ray $\overrightarrow{BB'} \perp L$ to cut L at B`



Generally

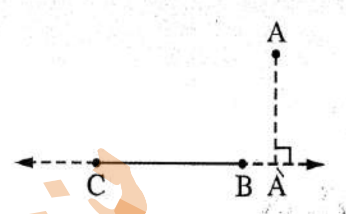
1 The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point and the straight line.

2 If the point lies on the straight line, its projection on it is the same point.

Remark

In the opposite figure :

The point $\grave{A}$ is the projection of the point A on the straight line $\overleftrightarrow{BC}$



② The projection of a line segment on a straight line

Generally

The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.

The shape	The line segment	Its projection on L	The relation
	$\overline{AB}$	$\overline{\grave{A}\grave{B}}$	$\grave{A}\grave{B} < AB$
	$\overline{AB}$	$\overline{\grave{A}\grave{B}}$	$\grave{A}\grave{B} < AB$
	$\overline{AB}$	$\overline{A\grave{B}}$	$A\grave{B} < AB$
	$\overline{AB}$	$\overline{\grave{A}\grave{B}}$	$\grave{A}\grave{B} = AB$
	$\overline{AB}$	The point C	$\grave{A}\grave{B}$ = zero

From the table, we notice that :

The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

③ The projection of a ray on a straight line

1 In the opposite figure :

$\overrightarrow{AB}$ is a given ray , L is a given straight line in the same plane. If A' is the projection of A on the straight line L , B' is the projection of B on the straight line L,
then the ray $\overrightarrow{A'B'}$ is the projection of the ray $\overrightarrow{AB}$ on the straight line L

If $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$ and if D' is the projection of D on the straight line L, then $D' \in \overrightarrow{A'B'}$, $D' \notin \overline{A'B'}$

i.e.

The projection of a ray on a straight line not perpendicular to it is a ray ⊂ this straight line.

2 In the opposite figure :

If $\overrightarrow{AB} \perp$ the straight line L, then the projection of $\overrightarrow{AB}$ on the straight line L is the point C

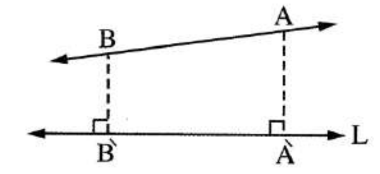
i.e.

The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.

④ The projection of a straight line on another straight line

1 In the opposite figure :

The projection of $\overleftrightarrow{AB}$ on the straight line L is the straight line $\overleftrightarrow{A'B'}$ or the straight line L

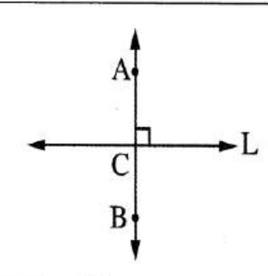


i.e.

The projection of a straight line on a straight line not perpendicular to it is a straight line.

2 In the opposite figure :

If $\overleftrightarrow{AB} \perp$ the straight line L , then the projection of $\overleftrightarrow{AB}$ on the straight line L is the point C

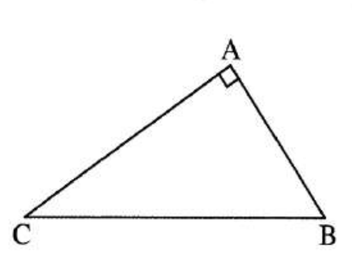


i.e.

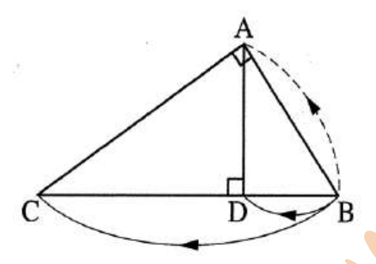
The projection of a straight line on a straight line perpendicular to it is the point of intersection of the two straight lines.

Euclidean theorem

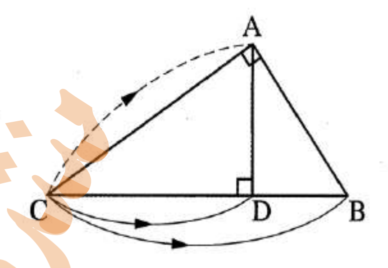
In the following , we write the summary of the relations of Pythagoras' theorem and Euclidean theorem :



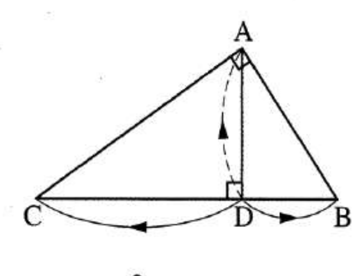
$$(BC)^2 = (AB)^2 + (AC)^2$$
$$(AB)^2 = (BC)^2 - (AC)^2$$
$$(AC)^2 = (BC)^2 - (AB)^2$$



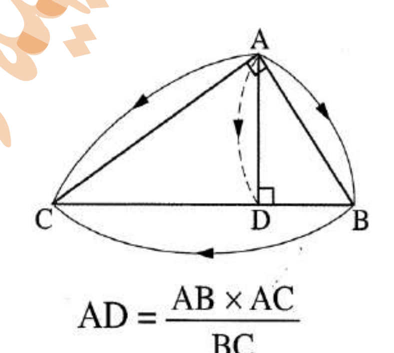
$$(BA)^2 = BD \times BC$$



$$(CA)^2 = CD \times CB$$



$$(DA)^2 = DB \times DC$$



$$AD = \frac{AB \times AC}{BC}$$

Classifying triangles according to their angles

Remarks

1. To determine the type of an angle in a triangle , we compare between the square length of the side opposite to it and the sum of squares lengths of the other two sides.
2. The greatest angle in measure in the triangle is opposite to the longest side.
3. In any triangle , there are two acute angles at least.

Remark

In any triangle (right , acute or obtuse-angled triangle) , we find that :
The length of any side of the triangle is greater than the difference between the lengths of the other two sides and less than the sum of their lengths.

i.e. If ABC is a triangle , then :

- $BC - AC < AB < BC + AC$
- $AB - AC < BC < AB + AC$
- $AB - BC < AC < AB + BC$

questions



(1) Complete the following:

1) The area of the triangle whose base length 10cm and height 6cm equals ……… cm^2.
2) Two triangles which have the same base and their vertices opposite to this base on a straight line parallel to the base are …….. in area.
3) The area of the rhombus whose diagonals 12 cm, 8 cm equals ………….. cm^2.
4) The median of a triangle divide it into two triangle …….. in the area,
5) The area of trapezium whose parallel base 6 cm, 10 cm and height 5 cm. equals ………….
6) If two triangles have equal areas and drawn on the same base and in one side of it then ………..
7) Surface of two parallelograms with common base and between two parallel lines ……………..
8) The median of a triangle divides its surface into ……..
9) Area of the parallelogram equals ………
10) Triangles of equal bases in length and lying between two parallel lines are equal in ……………
11) The area of the rhombus whose diagonals X cm, Y cm is …….
12) The area of the right angled triangle whose sides length of the right angle are 6 cm , 8 cm equals ……..
13) The area of the trapezium whose middle base 9 cm and height 6 cm equals ………

14) The measure of base angles of an isosceles trapezium are .....

15) The lengths of two adjacent sides in a parallelogram are 9 cm, 6 cm and the smallest height is 4cm then the length of the other height is ....

16) The height of trapezium whose parallel base are 5 cm, 7 cm and area of 42 cm^2 is ....

17) The area of rhombus whose perimeter is 20 cm and height 4 cm = ...

18) The length of the diagonal of a square of area 50 cm^2 equals .... cm .

19) The length of side of a square whose area equals the area of a rectangle with dimensions 9 cm , 16 cm = ...

20) The length of the middle base of a trapezium whose area = 30 cm^2 and height 5 cm equals ....

(2) Choose the correct answer:-

1) The length of the base of a triangle whose area 30 cm^2 and height 6 cm....

a) 5 b) 10 c) 15 d) 20

2) The length of the two adjacent sides in a parallelogram are 7 cm, 5 cm and the length of its smallest height is 4 cm then the area of the parallelogram equals .... cm^2.

a) 35 b) 25 c) 28 d) 49

3) The area of trapezium whose middle base length is 10 cm and height 8 cm equals .... cm^2.

a) 80 b) 60 c) 40 d) 20

4) The quadrilateral whose area equals half square of its diagonal is …………

a) parallelogram   b) rectangle   c) rhombus   d) square

5) The diagonals of an isosceles trapezium ………..

a) congruent   b) perpendicular

c) bisect each other   d) parallel

6) The area of rhombus whose diagonals length are 6 cm, 8 cm equals…..

a) 2 cm^2 b) 14 cm^2 d) 24 cm^2 d) 48 cm^2

7) The ratio between area of parallelogram and area of triangle if they have a common base and including between two parallel lines equals ……..

a) 1 : 2   b) 1 : 3   d) 2 : 1   d) 2 : 3

8) If the area of a square 18 cm^2 then length of its diagonal is …

a) 36   b) 12   c) 9   d) 6

9) If two triangles area equal in area and drawn on same base and in one side of it then their vertices lie on a straight line.

a) perpendicular to this base.   b) bisect this base

c) parallel to this base   d) intersects the base.

10) The quadrilateral whose area equals the square of its side length is…

a) parallelogram   b) rectangle

c) rhombus   d) square

11) The area of the rectangle whose dimensions 5 cm, 4 cm is ….

a) 9 cm^2 b) 10 cm^2 c) 20 cm^2 d) 40 cm^2

12) The side length of a square whose area equals the area of a parallelogram of base length 8 cm and corresponding height 4.5cm equals......

a) 6 cm b) 13 cm c) 18 cm d) 36 cm

13) The median of a triangle divides its surface into two triangles ....

a) congruent b) equals in area

c) isosceles d) right angles

14) The perimeter of the square whose area 81 cm^2 = ... cm.

a) 24 b) 8 c) 9 d) 36

15) If the area of a rhombus is 24 cm^2 and the length of one of its diagonal is 6 cm then the length of the other diagonal is ....

a) 4 cm b) 8 cm c) 10 cm d) 12 cm

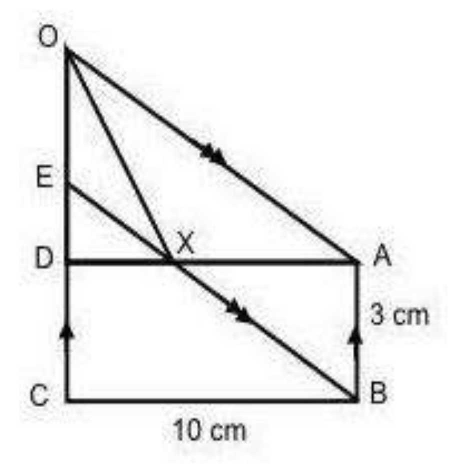
(3) Essay Questions:-

(1) In opposite figure :

ABCD is a rectangle, ABEO is a parallelogram,

AB = 3 cm, BC = 10 cm

Find with proof: the area of Δ AXO

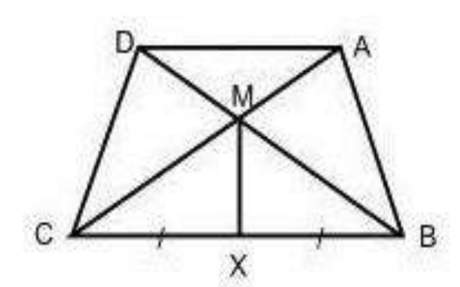


(2) In the opposite figure:

$\overline{AD} \parallel \overline{BC}$, X midpoint of $\overline{BC}$ prove that:

(i) Area of Δ AMB = area of Δ DMC

(ii) Area of shape ABXM = area of shape DCXM

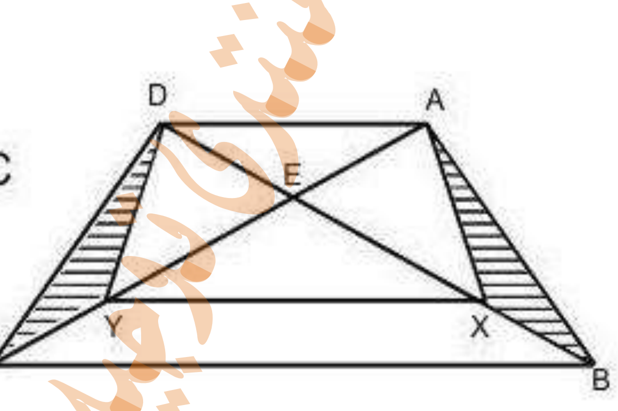


(3) The area of a trapezium is 88 cm^2, its height is 8 cm and the length of one of the two parallel base 10 cm, find the length of the other base.

(4) In the opposite figure:

$\overline{AD} \parallel \overline{BC}$ area of Δ AXB = area of Δ DYC

Prove that: $\overline{XY} \parallel \overline{AD}$



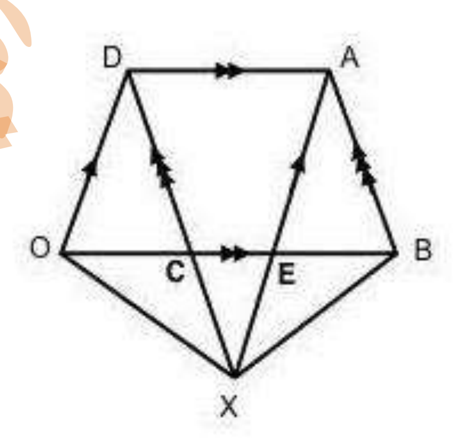
(5) In the opposite figure:

ABCD , AEOD area two parallelograms

$\overrightarrow{AE} \cap \overrightarrow{DC} = \{X\}$

Prove that

Area of Δ ABX equals area of Δ DOX

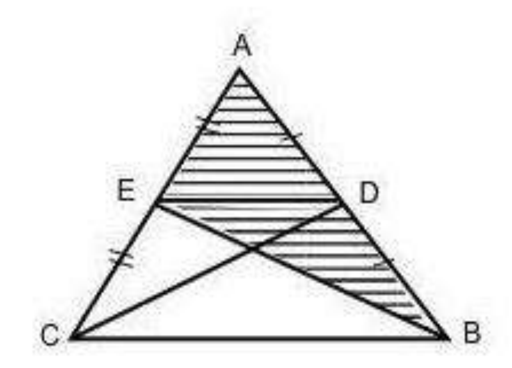


(6) Two pieces of land have equal areas, one of them has the shape of a square and the other has the shape of trapezium with two parallel bases of lengths 7 m, 11 m and height of 4m find the perimeter of the square land.

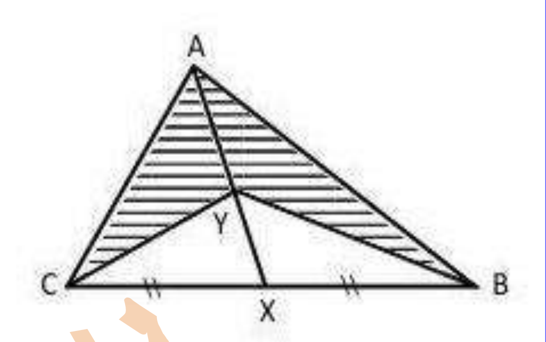
(7) In the opposite figure

If area of (Δ ADC) = are of (Δ AEB)

Prove that $\overline{DE} \parallel \overline{BC}$



(8) In the opposite figure:

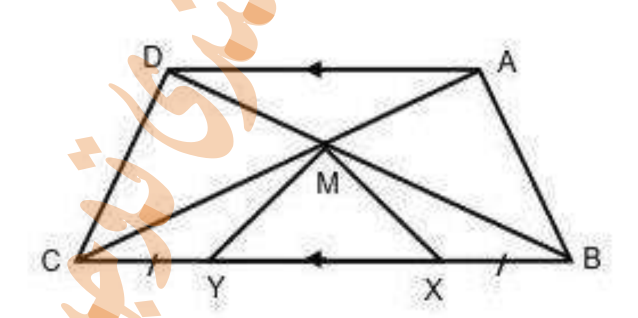


$\overline{AX}$ is a median in Δ ABC

, Y ∈ $\overline{AX}$, $\overline{BY}$, $\overline{CY}$ are drawn prove that

area of (Δ ABY) = area of (Δ ACY)

(9) In the opposite figure:



$\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ ∩ $\overline{BD}$ = {M}

X , Y ∈ $\overline{BC}$ such that BX = CY

Prove that:

area of shape ABXM = area of shape DCYM

(10) ABCD is a parallelogram in which $\overline{DE}$ ⊥ $\overline{BC}$, $\overline{DO}$ ⊥ $\overline{AB}$

if AB = 4 cm, BC = 6 cm , DE = 3 cm find the length of $\overline{DO}$

Part (2)

First : Complete the following:

1) If $\overline{AB} \perp \overline{BC}$ then the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ....

2) In Δ ABC if $(AB)^2 = (BC)^2 + (AC)^2$ then m ($\angle$ ....) = 90°

3) The two polygons are similar to a third are ....

4) The two triangles are similar if its corresponding angles are .... in measure.

5) ABC is a right angled triangle at B in which AB = 5 cm, BC = 12 cm then AC = ... cm.

6) The projection of a point which belongs to a straight line on this line is ....

7) In Δ ABC if $(AC)^2 + (AB)^2 < (BC)^2$ then angle A is ....

8) In Δ XYZ if $(ZX)^2 + (YZ)^2 > (XY)^2$ then angle Z is ....

9) In the opposite figure:

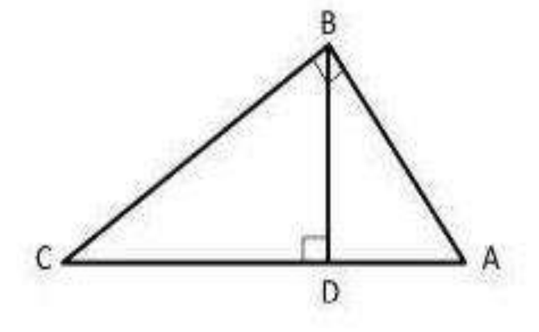
Δ ABC is right angle triangle at B, $\overline{BD} \perp \overline{AC}$

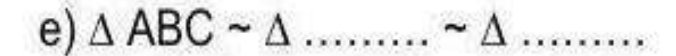
a) The projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ is

b) $(AB)^2$ = AD × ....

c) $(BD)^2$ = AD × ....

d) $(BC)^2$ = CD × ....

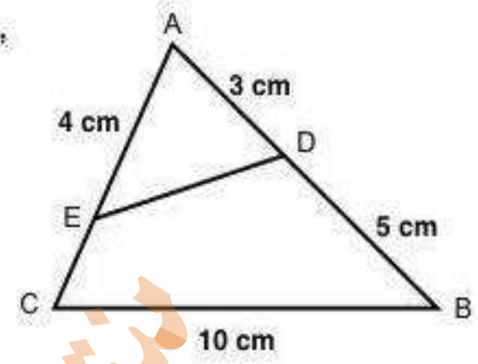


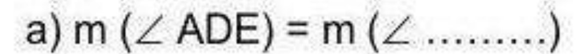

e) Δ ABC ~ Δ .... ~ Δ ....

10) In the opposite figure:

If Δ AED ~ Δ ABC, AD = 3 cm, AE = 4 cm,

BC = 10 cm , BD = 5 cm then





a) m ($\angle$ ADE) = m ($\angle$ ....)

b) m ($\angle$ BAC) = m ($\angle$ ....)

c) DE = ... cm

d) ED = ... cm

11) The area of a rectangle whose length of one of its dimensions = 12 cm, its diagonal = 13 cm equal ....

12) The triangle of side length 3 cm, 4 cm, 5 cm is .... angled triangle.

13) Two triangles are similar one of them has sides length 9 cm, 12 cm, 16 cm and the perimeter of the other 148 cm then side lengths of the other triangle are .... , ... , ...

Second: Choose the correct answer:

1) If Δ ABC ~ Δ DEO, AB = $\frac{1}{4}$ DE then the perimeter of Δ ABC equals .... the perimeter of Δ DEO.

a) 4 b) 2 c) $\frac{1}{2}$ d) $\frac{1}{4}$

2) The length of the projection of a given line segment .... the length of the original line segment.

a) $\geq$ b) > c) $\leq$ d) <

3) ABC is an obtuse angle triangle at A in which AB = 5 cm, BC = 8 cm then AC = ... cm

a) 5 b) 7 c) 8 d) 13

4) The triangle whose sides length are 3 cm, 4 cm, 5 cm its area = … cm^2

a) 12 b)10 c) 8 d) 6

5) If the ratio of enlargement between two similar triangles equals …………. then the two triangles are congruent.

a) 1 b) 2 c) 0.5 d) 0.25

6) Δ ABC in which $(AC)^2 = (BC)^2 - (AB)^2$ then angle A is ………..

a) acute b) right c) obtuse d) straight

7) The triangle whose sides length are 5 cm, 12 cm, 13 cm its area = ……….. cm^2

a) 30 b) 32.5 c) 78 d) 144

8) Δ ABC is obtuse angle triangle at B and AB = 3 cm, BC = 5 cm then AC = ………..

a) 8 cm b) 7 cm c) 15 cm d) 4 cm

9) In the two similar polygons their corresponding angles are ……… in measure.

a) equal b) difference c) proportional d) alternatives

10) The perpendicular segment drawn from the right angle of a triangle to the hypotenuse divides it to two triangles.

a) obtuse angle b) acute angle

c) equal's sides triangle d) similar

11) ABC is a triangle in which $\overline{AD} \perp \overline{BC}$ then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ………..

a) $\overline{BD}$ b) $\overline{DC}$ c) $\overline{AC}$ d) $\overline{AB}$

12) Δ ABC in which $(AB)^2 + (BC)^2 < (AC)^2$ then $\angle$ B is ………..

a) acute b) right c) obtuse d) reflex

13) The diagonal of a square whose area 50 cm^2 equals ....

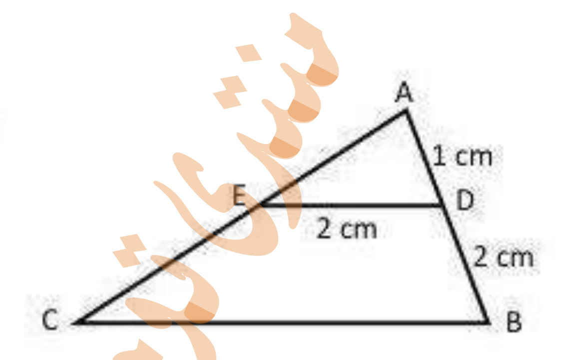
a) 10 cm b) 20 cm c) 30 cm d) 40 cm

14) Δ ABC in which $(AB)^2 = (AC)^2 + (BC)^2$, m (∠ B) = 40° then m (∠ A) = ...

a) 40° b) 50° c) 90° d) 130°

15) In the opposite figure:

If Δ ADE ~ Δ ABC then the length of $\overline{BC}$ in cm equals ....



a) 3 b) 4

c) 6 d) 8

Third: Essay question:

(1) Determine the type of the angle B in ΔABC in each of the following:

a) AB = 7 cm , BC = 12 cm , AC = 8 cm

b) AB = 5 cm , BC = 8 cm , AC = 11 cm

c) AB = 6 cm , BC = 3.6 cm , AC = 4.6 cm

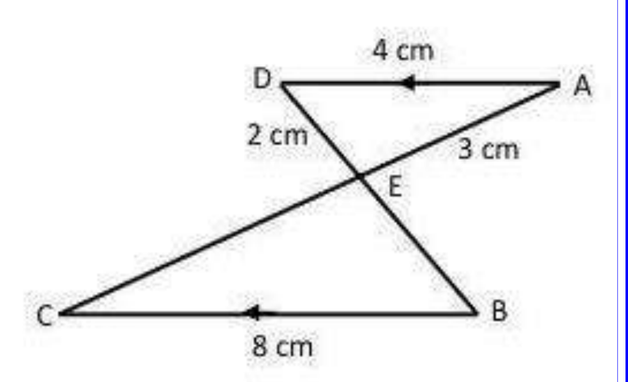
(2) In the opposite figure:

$\overline{AD} // \overline{BC}$, AD = 4 cm, BC = 8 cm ,

AE = 3 cm, ED = 2 cm

i) Prove that Δ AED ~ Δ CEB

ii) Find the perimeter of Δ EBC



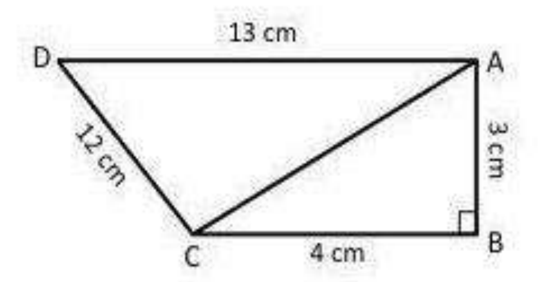
(3) In the opposite figure:

AB = 3 cm , BC = 4 cm ,

AD = 13 cm, CD = 12 cm

m (∠ B) = 90°

Prove that m (∠ ACD) = 90°



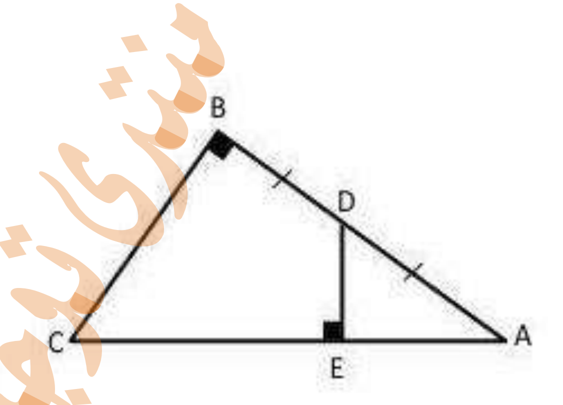
(4) In the opposite figure:

ABC is right angle triangle at B,

D is the midpoint

of $\overline{AB}$, $\overline{DE} \perp \overline{AC}$, AB = 8 cm, BC = 6 cm

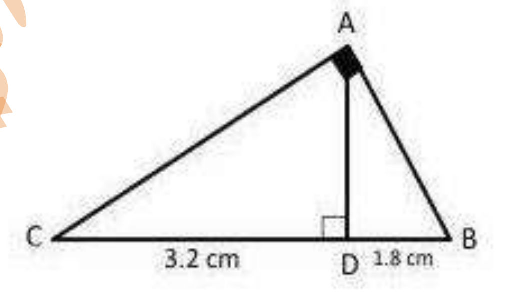
Find the length of $\overline{DE}$



(5) In the opposite figure:

ED = 1.8 cm, DC = 3.2 cm

Find the lengths of each $\overline{AC}$, $\overline{AD}$



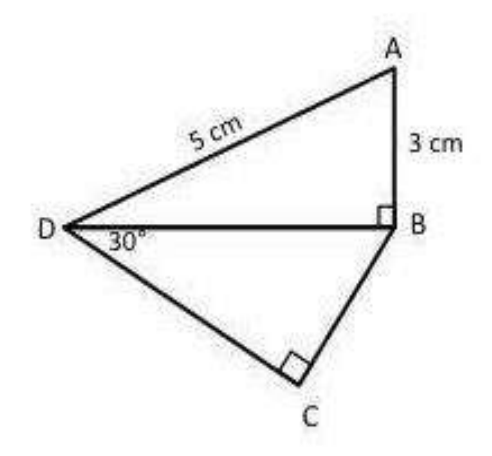
(6) In the opposite figure:

ABCD is quadrilateral in which

m (∠ ABD) = 90°, m (∠ BCD) = 90°,

m (∠ BDC) = 30°,

AB = 3 cm , AD = 5 cm find $\overline{BC}$



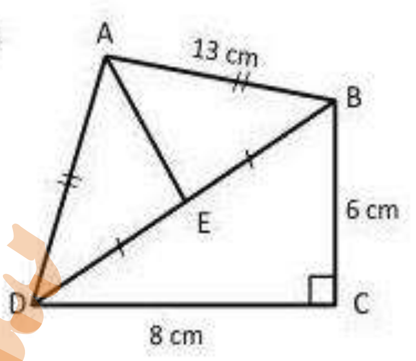
(7) In the opposite figure:

ABCD is a quadrilateral in which m (∠ C) = 90°

AB = AD = 13 cm, BC = 6 cm, CD = 8 cm

E is midpoint of $\overline{BD}$

Find the area of the shape ABCD



(8) In the opposite figure:

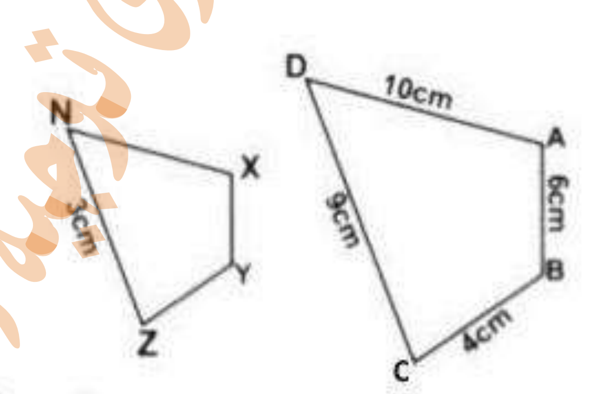
The polygon ABCD

is similar to the polygon XYZN ,

AB = 6 cm , BC = 4 cm ,

CD = 9 cm , DA = 10 cm

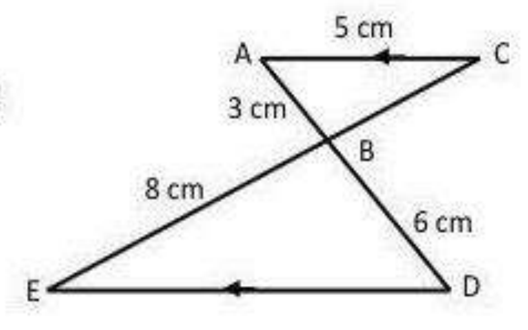
, ZN = 3 cm find the lengths of $\overline{XY}$, $\overline{YZ}$, $\overline{XN}$



(9) In the opposite figure:

i) Prove that Δ ABC is similar Δ DBE

ii) Find the length of $\overline{BC}$, $\overline{DE}$



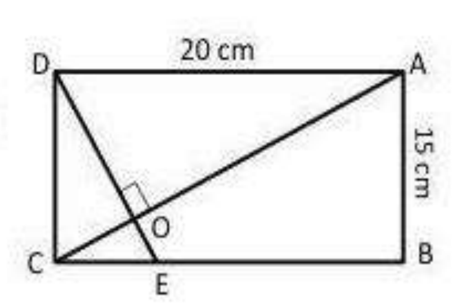
(10) In the opposite figure:

ABCD is a rectangle $\overline{DE} \perp \overline{AC}$

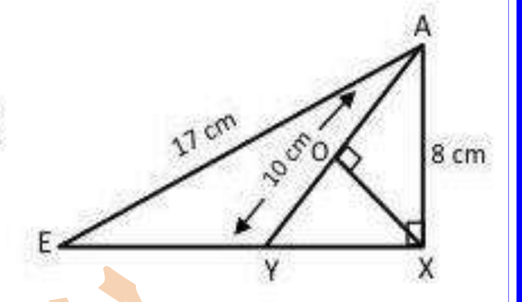
, DE intersect AC at O and intersect BC at E

If AB = 15 cm, AD = 20 cm

Find the lengths of each $\overline{AO}$, $\overline{CE}$



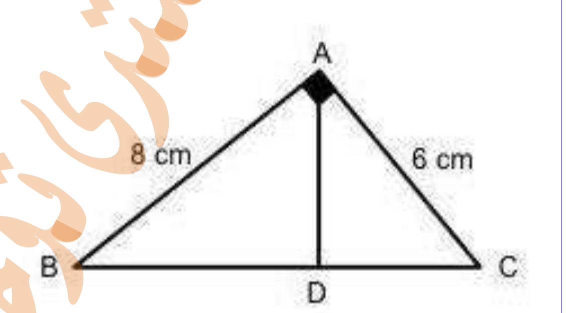
(11) In the opposite figure:



i) Find the length of projection of $\overline{AY}$ on $\overleftrightarrow{XE}$

ii) Find the length of $\overline{XO}$, $\overline{AO}$

(12) In the opposite figure:



Δ DBA ~ Δ ABC , m ($\angle$ BAC) = 90°

Prove that: $\overline{AD} \perp \overline{BC}$

Find BD if AB = 8 cm , AC = 6 cm

(13) A piece of land has a rectangle shape whose length twice its width and its area 200 meter square is drawn by a scale 1:200 find the dimensions of this land at the drawing.



Model Answers

Part (1)

(1) Complete:

1) 30 cm^2

2) equal.

3) 48 cm^2.

4) equal.

5) $\frac{1}{2}(6 + 10) \times 5 = 40\ cm^2$

6) Their vertices lie on a straight line parallel to this base.

7) one is carrying this base are equal in area.

8) Two triangular surface equal in area.

9) the length of the base X its corresponding height.

10) Area.

11) $\frac{1}{2}$ XY cm^2.

12) $\frac{1}{2} \times 6 \times 8 = 24\ cm^2$

13) $9 \times 6 = 54\ cm^2$

14) equal in measure

15) 6 cm.

16) the middle base = $\frac{1}{2}(5 + 7) = 6$ cm

$H = 42 \div 6 = 7$ cm.

17) $b = 20 \div 4 = 5$ cm

$A = 5 \times 4 = 20\ cm^2$

18) 10 cm.

19) A. of rectangle = $9 \times 16 = 144\ cm^2$

S. of square = $\sqrt{144} = 12$cm.

20) $30 \div 5 = 6$cm.

(2) Choose the correct answer:-

1) (b)	2) (c)	3) (a)	4) (d)
5) (a)	6) (c)	7) (c)	8) (d)
9) (c)	10) (d)	11) (c)	12) (d)
13) (b)	14) (d)	15) (b)	

(3)

(1) Proof: ∵ ABCD is a rectangle, ABEO is a parallelogram

▭ ABCD , ▱ ABEO have common base $\overline{AB}$

∴ Area of ▭ ABCD = Area of ▱ ABEO

∵ AB = 3 cm , BC = 10 cm

∴ Area of ▭ ABCD = 3 × 10 = 30 cm^2

∴ Area of ▱ ABEO = 30 cm^2

∵ In Δ AXO , ▱ ABEO have common base $\overline{AO}$

, $\overline{AO}$ // $\overline{BE}$

∴ Area of Δ AXO = $\frac{1}{2}$ area of ▱ ABEO

= $\frac{1}{2}$ x 30 = 15 cm

(2) Proof: ∵ $\overline{AD}$ // $\overline{BC}$

In Δ ACD , Δ ADB have common base $\overline{AD}$

∴ Area of Δ ACD = Area of Δ ADB (1)

subtracting A. of Δ AMD from (1)

∴ Area of Δ DMC = Area of Δ AMB (2)

∵ X midpoint of $\overline{BC}$

∴ Area of Δ MXC = Area of Δ MXB (3)

Adding (2) & (3)

∴ Area of the shape DCXM = Area of the shape ABXM

(3) Area of trapezium = $\frac{1}{2}(b_1 + b_2) \times h$

$$88 = \frac{1}{2}(10 + b_2) \times 8$$

$$b_2 = 12 \text{ cm}$$

(4) $\because \overline{AD} \parallel \overline{BC}$

In Δ ADB , Δ ADC have common base $\overline{AD}$

∴ Area of Δ ADB = Area of Δ ADC (1)

∵ Area of Δ AXB = Area of Δ DYC (2)

subtracting (2) from (1)

∴ Area of Δ ADX = Area of Δ AYD (3)

have a common base $\overline{AD}$

$\therefore \overline{XY} \parallel \overline{AD}$

(5) ∵ ABCD , AEOD are two parallelogram

, $\overline{AD}$ is a common base

∴ Area of ▱ ABCD = Area of ▱ AEOD (1)

subtracting Area of the figure AECD from (1)

∴ Area of Δ ABE = Area of Δ DCO (2)

∴ OC = EB

∵ in Δ XCO , Δ XEB have common vertex X

, EB = CO

∴ Area of Δ XBE = Area of Δ XCO (3)

Adding (2) & (3)

∴ Area of Δ ABX = Area of Δ DOX

(6) Area of trapezium = $\frac{1}{2}(b_1 + b_2) \times h$

$= \frac{1}{2}(7 + 11) \times 4$

$= 36 \text{ cm}^3.$

Area of square = 36 cm^3.

$S = \sqrt{36} = 6$ cm.

Perimeter of square = $6 \times 4 = 24 \text{ cm}^2$

(7) Proof: ∵ Area of Δ ADC = Area of Δ AEB

subtracting Area of Δ ADE from both side

∴ Area of Δ EDC = Area of Δ DEB

, $\overline{ED}$ is a common base

∴ $\overline{ED} \parallel \overline{BC}$

(8) Proof: ∵ In Δ ABC

X is midpoint

∴ A. of Δ ABX = A. of Δ AXC (1)

∵ In Δ YBC

X is midpoint

∴ A. of Δ YBX = A. of Δ YXC (2)

subtracting (2) from (1)

∴ A. of Δ ABY = A. of Δ ACY .

(9) Proof: ∵ In Δ ABD , Δ ACD

$\overline{AD} \parallel \overline{BC}$, $\overline{AD}$ is a common base .

∴ Area of Δ ABD = Area of Δ ADC (1)

By subtracting Area of Δ AMD from both side

∴ Area of Δ AMB = Area of Δ DMC (2)

∵ Δ MXB , Δ MYC

M is a common vertex, XB = YC

∴ Area of Δ MXB = A. of Δ MYC (3)

Adding (2) & (3)

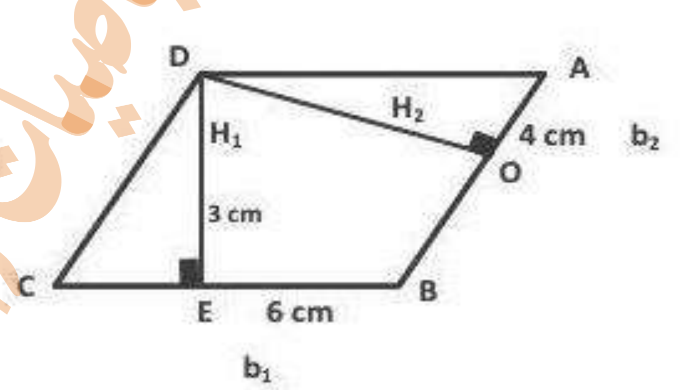
∴ Area of shape ABXM = Area of shape DCYM

(10) Area of parallelogram

$= b_1 \times h_1 = 3 \times 6 = 18 \text{ cm}^2$

$A = b_2 \times h_2 = 4 \times h_2 = 18 \text{ cm}^2$

h_2 (DO) = 18 ÷ 4 = 4.5 cm



Part (2)

First: Complete:

1) $\overline{BC}$ 2) (∠ C) 3) similar
4) equal 5) 13 cm 6) the same point
7) obtuse 8) acute
9) a) $\overline{AC}$ b) AC c) DC d) $\overline{CA}$ e) Δ ADB - Δ BDC
10) a) m (∠ ACB) b) m (∠ EAD) c) 5 cm d) 2 cm
11) 60 cm^2 12) right
13) 36 cm , 48 cm , 64 cm

Second: Choose:

1) d	2) c	3) a	4) d	5) a
6) b	7) a	8) b	9) a	10) d
11) a	12) a	13) a	14) b	15) c

Third: Essay Question

(1) a) obtuse b) obtuse c) obtuse

(2) ∵ $\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ & $\overline{DB}$ are transversals

∴ m (∠ D) = m (∠ B)

m (∠ A) = m (∠ C) alternate angles → (1)

∵ $\overleftrightarrow{DB} \cap \overleftrightarrow{AC}$ = { E }

∴ m (∠ DEA) = m (∠ BEC) V.O.A → (2)

From (1) & (2)

∴ Δ ADE ~ Δ CBE

$$\therefore \frac{AD}{CB} = \frac{DE}{BE} = \frac{AE}{CE} = \frac{P.of\,\Delta\,ADE}{P.of\,\Delta\,CBE}$$

$\therefore \frac{4}{8} = \frac{2}{BE} = \frac{3}{CE} = \frac{4+2+3}{\text{P.of } \Delta\, CBE}$

P. of Δ CBE = $\frac{9 \times 8}{4}$ = 18 cm

(3) In Δ ABC: ∵ m (∠ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras)

$AC = \sqrt{(3)^2 + (4)^2} = 5$ cm

In Δ ACD

$\because (AD)^2 = (13)^2 = 169$,

$(AC)^2 = 25$, $(CD)^2 = 144$

$\therefore (AD)^2 = (AC)^2 + (CD)^2$

∴ m (∠ ACD) = 90° (converse of Pythagoras theory)

(4) In Δ ABC: ∵ (∠ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 64 + 36 = 100$

∴ AC = 10 cm

, ∵ D is the midpoint of $\overline{AB}$

∴ AD = DB = 4 cm

In ΔΔ AED , ABC

m (∠ AED) = m (∠ B) = 90° (given)

, ∠ A is common

∴ m (∠ ADE) = m (∠ ACB)

∴ Δ AED ~ Δ ABC

$\therefore \frac{DE}{CB} = \frac{AD}{AC}$, $\therefore \frac{DE}{6} = \frac{4}{10}$

$\therefore DE = \frac{6 \times 4}{10} = 2.4$ cm

(5) In Δ ABC :

$\because$ m ($\angle$ A) = 90° , $\overline{AD} \perp \overline{CB}$

$\therefore$ $(AC)^2$ = CD × CB = 3.2 × 5 = 16 (Euclidean theorem)

AC = 4 cm

$(AD)^2$ = DB × DC = 1.8 × 3.2 = 5.76

AD = 2.4 cm

(6) In Δ ABD: $\because$ m ($\angle$ B) = 90°

$\therefore$ (BD) = $\sqrt{(5)^2 - (3)^2}$ = 4 cm (Pythagoras theorem)

In Δ BCD: $\because$ m ($\angle$ C) = 90° , m ($\angle$ CDB) = 30°

$\therefore$ CB = $\frac{1}{2}$ BD = $\frac{1}{2}$ × 4 = 2 cm

(7) In Δ BCD: $\because$ m ($\angle$ C) = 90°

$\therefore$ BD = $\sqrt{(BC)^2 + (CD)^2} = \sqrt{(6)^2 + (8)^2}$ = 10 cm

In Δ ABD: E is a midpoint of $\overline{BD}$, AB = AD (Pythagoras Theorem)

$\therefore$ AE $\perp$ BD , EB = 5 cm

$\therefore$ AE = $\sqrt{(AB)^2 - (EB)^2} = \sqrt{(13)^2 - (5)^2}$ = 12 cm

$\because$ The area of the quadrilateral ABCD =

Area of Δ BCD + Area of Δ ABD

$\therefore$ Area = $\frac{1}{2}$ × DC × BC + $\frac{1}{2}$ × BD × AE

= $\frac{1}{2}$ × 8 × 6 + $\frac{1}{2}$ × 10 × 12 = 24 + 60 = 84 cm^2

(8) $\because$ Polygon ABCD ~ Polygon XYZN

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZN} = \frac{AD}{XN}$

$\frac{6}{XY} = \frac{4}{YZ} = \frac{9}{ZN} = \frac{10}{XN}$

XY = 2 cm , YZ = $1\frac{1}{3}$ cm , XN = $3\frac{1}{3}$ cm

(9) $\because \overline{AC} // ED$, $\overleftrightarrow{AD}$ & $\overleftrightarrow{CE}$ are transversals

$\therefore m(\angle A) = m(\angle D)$

$m(\angle C) = m(\angle E)$ alternate angles → (1)

$\because \overleftrightarrow{AD} \cap \overleftrightarrow{CE} = \{B\}$, $\therefore m(\angle ABC) = m(\angle EBD)$ V.O.A → (2)

From (1) & (2)

$\therefore \Delta ABC \sim \Delta DBE$

$\therefore \frac{AB}{DB} = \frac{BC}{BE} = \frac{CA}{ED} = \frac{3}{6} = \frac{BC}{8} = \frac{5}{ED}$,

BC = 4 cm , ED = 10 cm

(10) In ΔABC: $\because (\angle B) = 90°$

$\therefore AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(15)^2 + (20)^2} = 25$ cm (Pythagoras)

In ΔADC: $\because (\angle D) = 90°$

$\therefore (DA)^2 = AO \times AC$ (Euclidean Theorem)

$\therefore AO = \frac{(20)^2}{25} = 16$ cm

$\therefore DO = \frac{DA \times DC}{AC} = \frac{20 \times 15}{25} = 12$ cm

$\because \Delta DCE$ is right angled at C , $\overline{CO} \perp \overline{DE}$

$\therefore (CD)^2 = DO \times DE \rightarrow DE = \frac{(15)^2}{12} = 18.75$ cm

OE = 18.75 - 12 = 6.75 cm

$(CE)^2 = EO \times ED = 6.75 \times 18.75 = 126.5625$ cm^2

CE = 11.25 cm

(11) $\because \overline{XY}$ is the projection of $\overline{AY}$ on $\overleftrightarrow{XE}$, Δ AXY is right angled

$\therefore (XY)^2 = (AY)^2 - (AX)^2 = 100 - 64 = 36$, XY = 6 cm

$\because \overline{XD} \perp \overline{AY}$, $XO = \frac{AX \times XY}{AY} = \frac{6 \times 8}{10} = 4.8$ cm

$(AX)^2 = AF \times AY$, AF = 6.4 cm

(12) $\because \Delta$ ABC is right angled at A , $\therefore BC = \sqrt{8^2 + 6^2} = 10$ cm

$\because \Delta DBA \sim \Delta ABC$, $\therefore m(\angle BDA) = m(\angle BAC) = 90°$

$\therefore \overline{AD} \perp \overline{BC}$, $\therefore (BA)^2 = BD \times BC$, $BD = \frac{64}{10} = 6.4$ cm

(13) Let the real length be = 2x , width = x

$A = L \times w = 2x \times x = 2x^2 = 200$ m → x = 10 cm , 2x = 20 m

Length in drawing = $\frac{2000 \times 1}{200} = 10$ cm   D.L : R.L

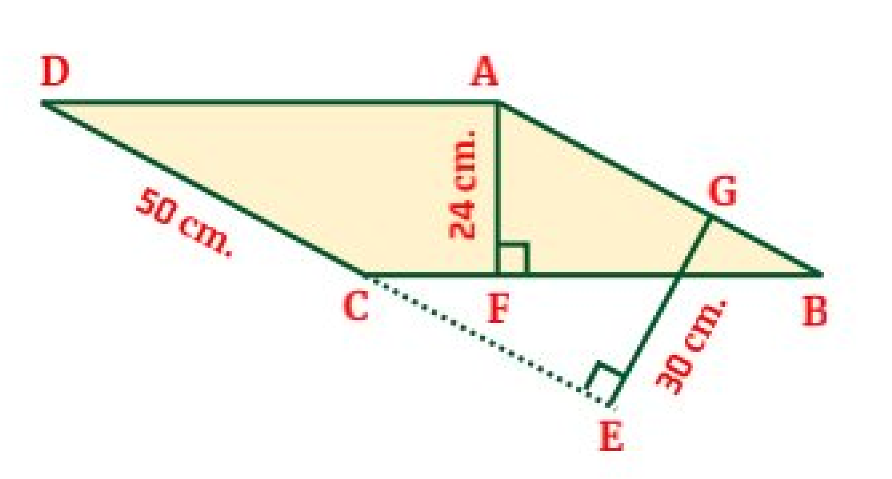
Width in drawing = $\frac{1000 \times 1}{200} = 5$ cm   1 : 200

1 AREAS



1 In the opposite figure :

ABCD is a parallelogram in which DC = 50 cm. , E ∈ $\overrightarrow{DC}$ where $\overrightarrow{GE} \perp \overrightarrow{DC}$, $\overline{AF} \perp \overline{BC}$, AF = 24 cm. , GE = 30 cm. , Find :



1. The area of the parallelogram
2. the length of $\overline{AD}$

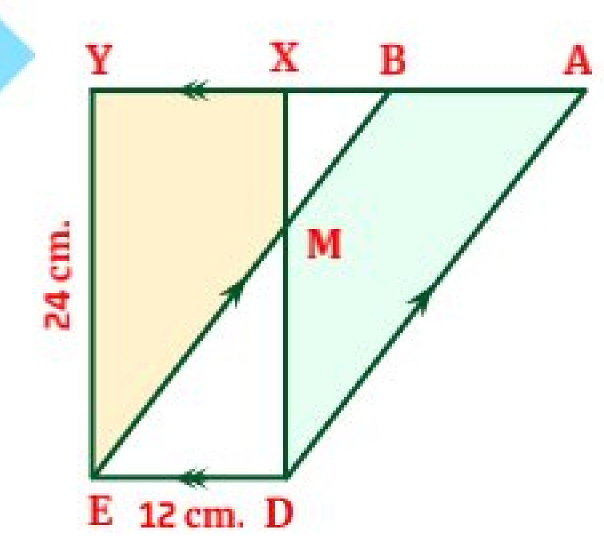
SOLUTION

1. the area of ▱ = AB × GE = 50 × 30 = 1500 cm^2
2. ∵ the area of ▱ = BC × AF = 24 BC = 1500 cm^2

   ∴ BC = 1500 ÷ 24 = 62.5 cm.

2 In the opposite figure :

$\overleftrightarrow{AB}$ // $\overleftrightarrow{DE}$, X and Y ∈ $\overleftrightarrow{AB}$, XDEY is a rectangle and $\overline{AD}$ // $\overline{BE}$



1. Prove that : the area of the figure ABMD = the area of the figure XYEM
2. Find : the area of the figure ABED
3. If : AD = 30 cm. Find : the length of the perpendicular from B to $\overline{AD}$.

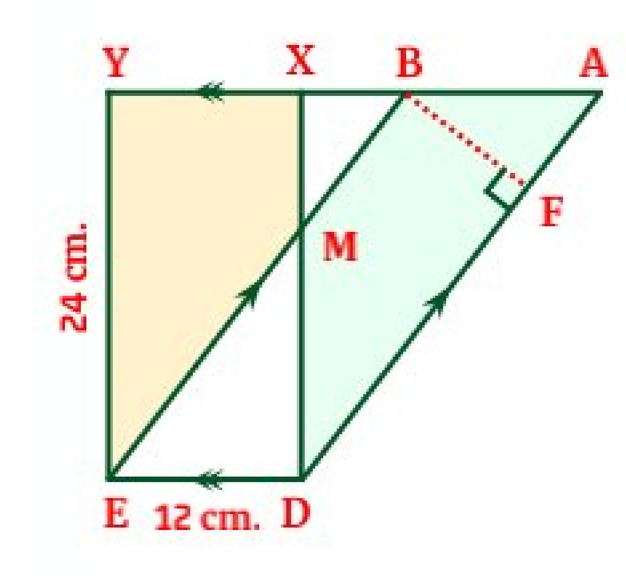
SOLUTION

1. ∵ $\overline{AD}$ // $\overline{BE}$ and $\overline{AB}$ // $\overline{DE}$ ∴ ABED is a parallelogram.

   ▱ ABED and ▭ XYED in which { $\overline{DE}$ is a common base , $\overline{DE}$ // $\overleftrightarrow{AY}$ }

   ∴ The area of ▱ ABED = the area of ▭ XYED   Subtracting the area of △ MED from the two sides

   ∴ the area of the figure ABMD = the area of the figure XYEM



2. ∵ The area of ▭ XYED = length × width = 24 × 12 = 288 cm^2

   ∴ The area of ▱ ABED = 288 cm^2

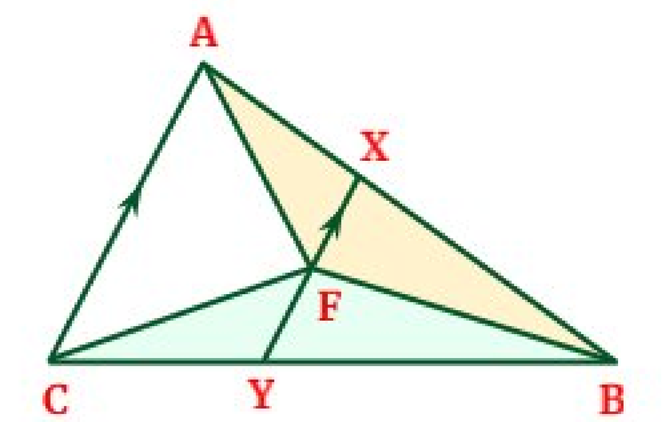
3. The area of ▱ ABED = AD × BF

   ∴ 30 × BF = 288   ∴ BF = 288 ÷ 30 = 9.6 cm.

3 In the opposite figure :

$\overline{AC}$ // $\overline{XY}$ and F is the midpoint of $\overline{XY}$.

Prove that : the area of △ ABF = the area of △ CBF



SOLUTION

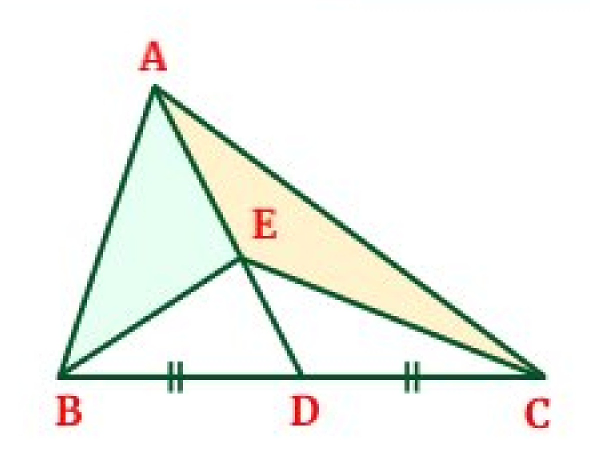
In △ AFX and △ CFY { FX = FY , $\overline{XY}$ // $\overline{AC}$ and F ∈ $\overline{XY}$ }

∴ the area of △ AFX = the area of △ CFY   ①

∵ $\overline{BF}$ is a midpoint in △ BXY   ∴ the area of △ BFX = the area of △ BFY   ②

Adding ① and ② we deduce : the area of △ ABF = the area of △ CBF

4 **In the opposite figure :**



ABC is a triangle with a median $\overline{AD}$, E ∈ $\overline{AD}$, draw $\overline{BE}$ and $\overline{CE}$

Prove that : The area of △ ABE = the area of △ ACE

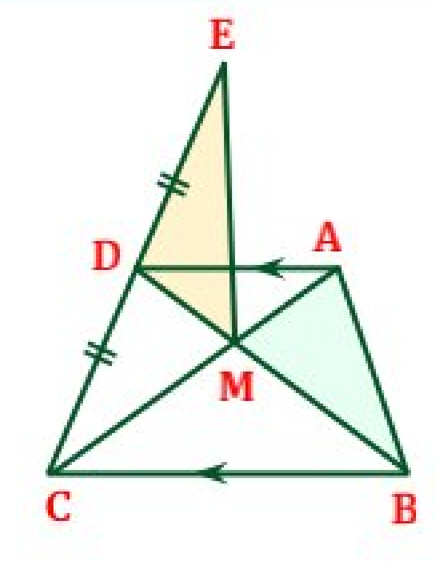
SOLUTION

∵ $\overline{AD}$ is a midpoint in △ ABC ∴ the area of △ ABD = the area of △ ACD ①

∵ $\overline{ED}$ is a midpoint in △ EBC ∴ the area of △ EBD = the area of △ ECD ②

Subtracting ② from ① we deduce : the area of △ ABE = the area of △ ACE

5 **In the opposite figure :**



$\overline{AD}$ // $\overline{BC}$ and $\overline{AC}$ ∩ $\overline{BD}$ = { M } , D is a midpoint of $\overline{EC}$

Prove that : The area of △ MDE = the area of △ AMB

SOLUTION

In △ ABC and △ DBC { $\overline{CB}$ is a common base , $\overline{AD}$ // $\overleftrightarrow{BC}$ }

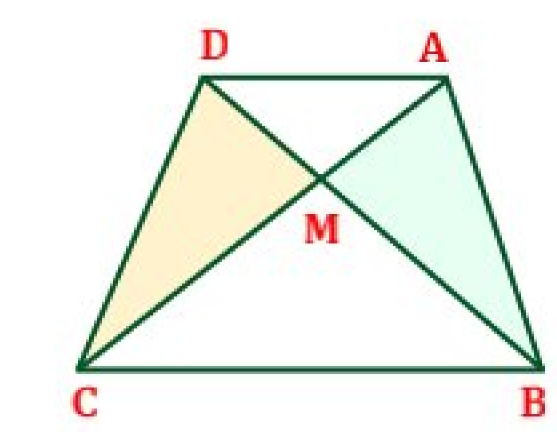
∴ the area of △ ABC = the area of △ DBC   Subtracting the area of △ MCB from the two sides

∴ the area of △ AMB = the area of △ DMC ①

∵ $\overline{MD}$ is a midpoint in △ CME ∴ the area of △ EMD = the area of △ DMC ②

From ① and ② we deduce : The area of △ MDE = the area of △ AMB

6 **In the opposite figure :**



ABCD is a quadrilateral , its diagonals intersect at M

, and the area of △ ABM = the area of △ DCM

Prove that : $\overline{AD}$ // $\overline{BC}$

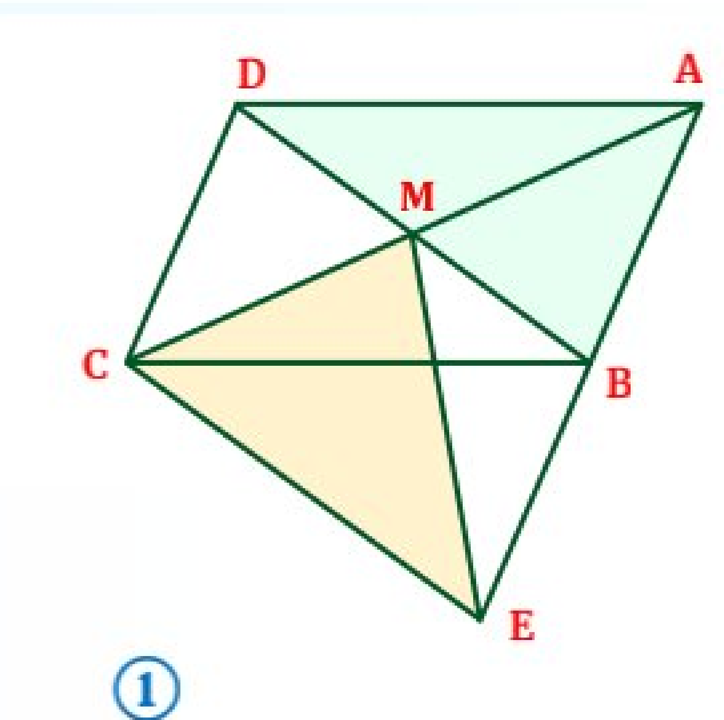
SOLUTION

∵ the area of △ ABM = the area of △ DCM   Adding the area of △ MCB to both sides

∴ the area of △ ABC = the area of △ DBC , but they have the base $\overline{BC}$ and on the same side of it.

∴ $\overline{AD}$ // $\overline{BC}$

7 **In the opposite figure :**



ABCD and BECD are two parallelogram , where $\overline{AC}$ ∩ $\overline{BD}$ = { M }

Prove that : the area of △ ABD = the area of △ MEC

SOLUTION

In ▱▱ ABCD and BECD { $\overline{CD}$ is a common base , $\overline{CD}$ // $\overline{AB}$, E ∈ $\overrightarrow{AB}$ }

∴ the area of ▱ ABCD = the area of ▱ BECD ①

In △ ABD and ▱ ABCD { $\overline{AB}$ is a common base , $\overline{AB}$ // $\overline{CD}$ }

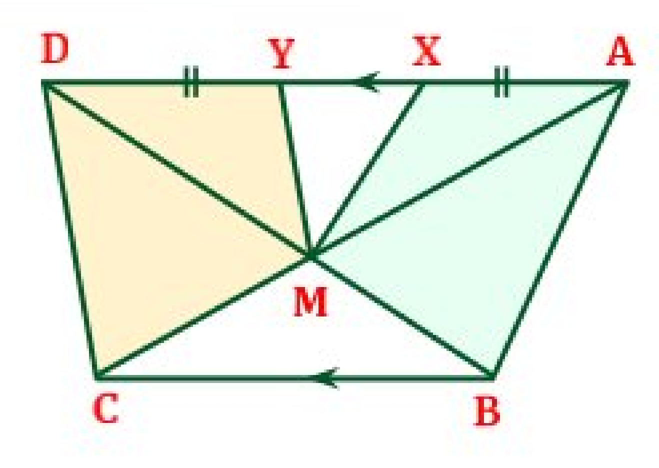
∴ the area of △ ABD = $\frac{1}{2}$ the area of ▱ ABCD ②

In $\triangle$ EMC and ▱ BECD { $\overline{CE}$ is a common base , $\overline{CE}$ // $\overline{BD}$, M ∈ $\overline{BD}$ }

∴ the area of $\triangle$ ABD = $\frac{1}{2}$ the area of ▱ BECD ③

From ① , ② and ③ we deduce : the area of $\triangle$ ABD = the area of $\triangle$ MEC

8 In the opposite figure :



ABCD is a quadrilateral whose diagonals intersect at M , $\overline{AD}$ // $\overline{BC}$

X and Y ∈ $\overline{AD}$ such that AX = DY.

Prove that : the area of the figure ABMX = the area of the figure DCMY

SOLUTION

In $\triangle$ ABC and $\triangle$ DBC { $\overline{CB}$ is a common base , $\overline{AD}$ // $\overleftrightarrow{BC}$ }

∴ the area of $\triangle$ ABC = the area of $\triangle$ DBC Subtracting the area of $\triangle$ MCB from the two sides

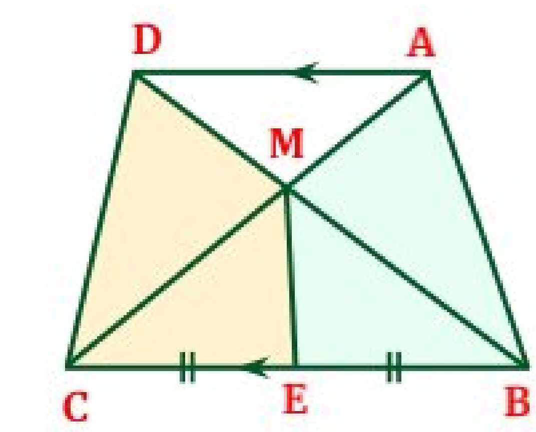
∴ the area of $\triangle$ AMB = the area of $\triangle$ DMC ①

In $\triangle$ $\triangle$ AMX and DMY { DY = AX and M is a common vertex }

∴ the area of $\triangle$ AMX = the area of $\triangle$ DMY ②

Adding ① and ② we deduce : the area of the figure ABMX = the area of the figure DCMY

9 In the opposite figure :



$\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ ∩ $\overline{BD}$ = { M } , E is a midpoint of $\overline{BC}$

Prove that : the area of the figure ABEM = the area of the figure DMEC

SOLUTION

In $\triangle$ ABC and $\triangle$ DBC { $\overline{CB}$ is a common base , $\overline{AD}$ // $\overleftrightarrow{BC}$ }

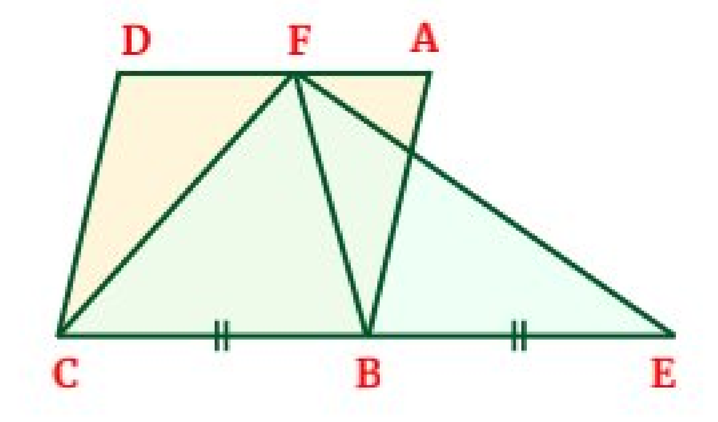
∴ the area of $\triangle$ ABC = the area of $\triangle$ DBC Subtracting the area of $\triangle$ MCB from the two sides

∴ the area of $\triangle$ AMB = the area of $\triangle$ DMC ①

∵ $\overline{ME}$ is a midpoint in $\triangle$ MBC ∴ the area of $\triangle$ MEB = the area of $\triangle$ MEC ②

Adding ① and ② we deduce : the area of the figure ABEM = the area of the figure DMEC

10 In the opposite figure :



ABCD is a parallelogram , E ∈ $\overrightarrow{CB}$, where BC = BE

Prove that : The area of $\triangle$ EFC = the area of ▱ ABCD

SOLUTION

In $\triangle$ FBC and ▱ ABCD : { $\overline{CB}$ is a common base , $\overline{CB}$ // $\overline{AD}$, F ∈ $\overline{AD}$ }

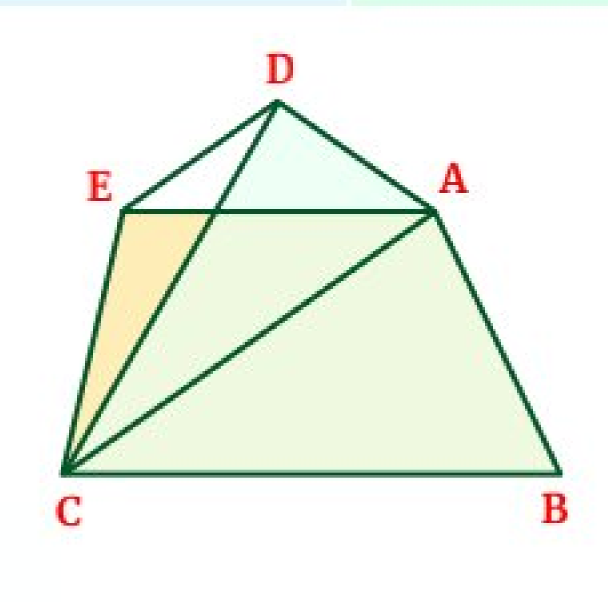
∴ The area of $\triangle$ FBC = $\frac{1}{2}$ the area of ▱ ABCD ①

In $\triangle$ $\triangle$ FCB and FBE { CB = BE and F is a common vertex }

∴ The area of $\triangle$ FCB = the area of $\triangle$ FBE ②

From ① and ② we deduce : The area of $\triangle$ EFC = the area of ▱ ABCD

11 **In the opposite figure :**



The area of the figure ABCD = the area of the figure ABCE

Prove that : $\overline{DE} // \overline{AC}$

SOLUTION

$\because$ The area of the figure ABCD = the area of the figure ABCE

Subtracting the area of $\triangle$ ABC from the two sides

$\therefore$ The area of $\triangle$ ACD = the area of $\triangle$ ACE , but they have the base $\overline{AC}$ and on the same side of it.

$\therefore$ $\overline{DE} // \overline{AC}$

12 **In the opposite figure :**

$\overline{AD} // \overline{BC}$, $\overline{AE} \cap \overline{BD} = \{ M \}$, the area of $\triangle$ AMB = the area of $\triangle$ EMC

Prove that : $\overline{ME} // \overline{DC}$

SOLUTION

In $\triangle$ ABE and $\triangle$ DBE { $\overline{EB}$ is a common base , $\overline{AD} // \overline{EB}$ }

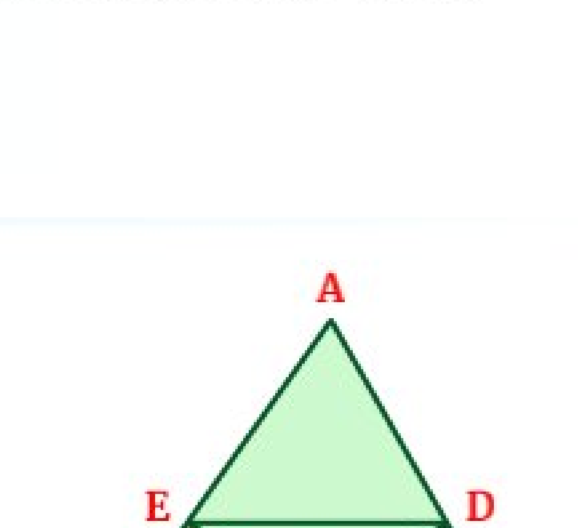
$\therefore$ the area of $\triangle$ ABE = the area of $\triangle$ DBE Subtracting the area of $\triangle$ MEB from the two sides

$\therefore$ the area of $\triangle$ AMB = the area of $\triangle$ DME , but the area of $\triangle$ AMB = the area of $\triangle$ EMC (given)

$\therefore$ the area of $\triangle$ DME = the area of $\triangle$ EMC , but they have the base $\overline{ME}$ and on the same side of it.

$\therefore$ $\overline{ME} // \overline{DC}$

13 **In the opposite figure :**



If the area of $\triangle$ ADC = the area of $\triangle$ AEB

Prove that : $\overline{DE} // \overline{BC}$

SOLUTION

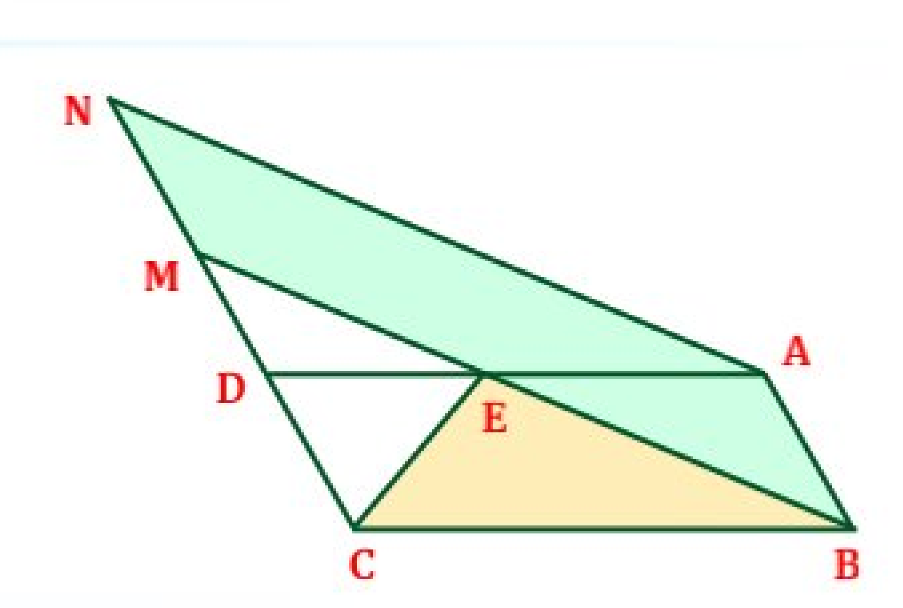
$\because$ the area of $\triangle$ ADC = the area of $\triangle$ AEB

Subtracting the area of $\triangle$ ADE from the two sides

$\therefore$ The area of $\triangle$ DBE = the area of $\triangle$ DCE , but they have the base $\overline{ED}$ and on the same side of it.

$\therefore$ $\overline{DE} // \overline{BC}$

14 **In the opposite figure :**



ABCD and ABMN are two parallelogram , M $\in \overrightarrow{CD}$

Prove that : The area of $\triangle$ EBC = $\frac{1}{2}$ the area of ▱ ABMN

SOLUTION

In ▱▱ ABCD and ABMN { $\overline{AB}$ is a common base , $\overline{AB} // \overline{CD}$, M , N $\in \overrightarrow{CD}$ }

$\therefore$ The area of ▱ ABCD = The area of ▱ ABMN ①

In $\triangle$ EBC and ▱ ABCD : { $\overline{CB}$ is a common base , $\overline{CB} // \overline{AD}$, F $\in \overline{AD}$ }

The area of △ EBC = $\frac{1}{2}$ the area of ▱ ABCD ②

From ① and ② we deduce : The area of △ EFC = the area of ▱ ABCD

15 The area of a trapezium is 88 cm^2, its height is 8 cm. and the length of one of the two parallel bases is 10 cm. , Find the length of the other base.

SOLUTION

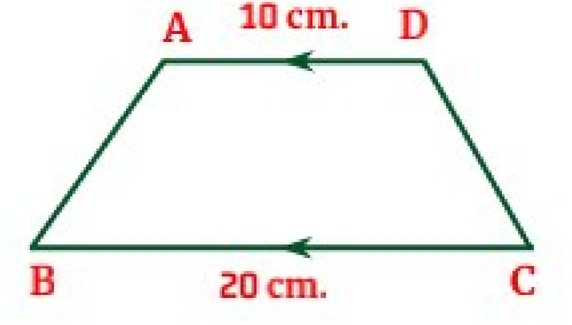
Let the length of other base = x cm.

∵ The area of the trapezium = $\frac{1}{2}(10 + x) \times 8$ ∴ $88 = 4(10 + x)$ (divide by 4)

∴ $x + 10 = 22$ ∴ $x = 22 - 10 = 12$ cm.

16 ABCD is a trapezium in which : $\overline{AD} \,//\, \overline{BC}$, if BC = 2 AD = 20 cm. and its area = 180 cm^2. Find its height.

SOLUTION



∵ 2 AD = 20 ∴ AD = 20 ÷ 2 = 10 cm.



∵ The area of the trapezium = $\frac{1}{2}(10 + 20) \times h$ ∴ 180 = 15 h (divide by 15)

∴ h = 180 ÷ 15 = 12 cm.

17 The area of a trapezium is 180 cm^2, its height is 9 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3 : 5.

SOLUTION

Let the length of the two bases are 3 x and 5 x

∵ The area of the trapezium = $\frac{1}{2}(3x + 5x) \times 9$ ∴ $180 = \frac{1}{2}(8x) \times 9$

∴ 180 = 36 x ∴ x = 180 ÷ 36 = 5

∴ The two bases are 15 cm. and 25 cm.

18 A rhombus with diagonal lengths are 12 cm. and 10 cm. and its height 8 cm. Find its perimeter.

SOLUTION

The area of the Rhombus = $\frac{1}{2}$ × first diagonal × second diagonal = $\frac{1}{2} \times 12 \times 10 = 60$ cm^2

Side length = Area ÷ height = 60 ÷ 8 = 7.5 cm.

The perimeter = side length × 4 = 7.5 × 4 = 30 cm.

19 Find the area of the rhombus whose perimeter is 52 cm. and the length of one of its diagonals is 10 cm.

SOLUTION

Side length = Perimeter ÷ 4 = 52 ÷ 4 = 13 cm.

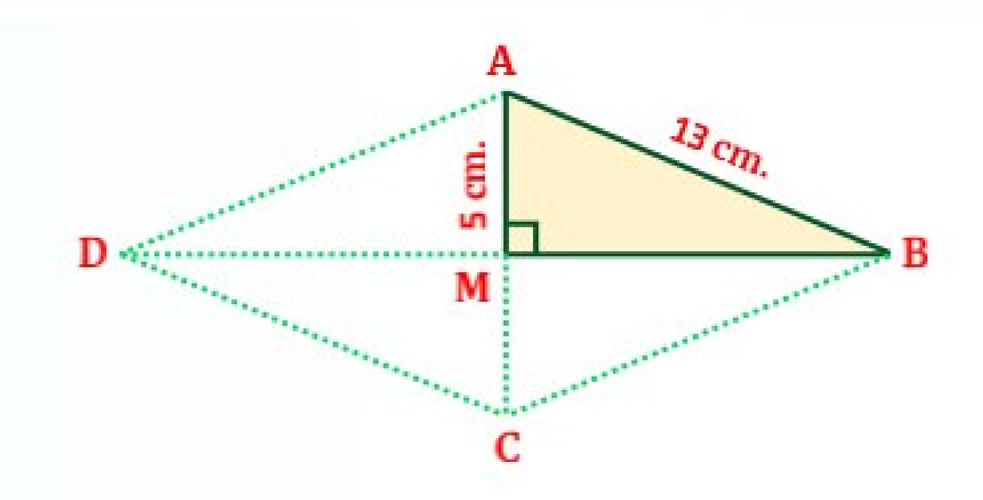
By drawing the rhombus ABCD and its two diagonal intersect at M we deduce :

$\because$ $\triangle$ AMB is a right-angled triangle at M , AM = 10 ÷ 2 = 5 cm.

$\therefore$ $(MB)^2 = (13)^2 - (5)^2 = 169 - 25 = 144$ $\therefore$ MB = 12 cm.

$\therefore$ DB = 12 × 2 = 24 cm.

$\therefore$ the area of the rhombus $= \frac{1}{2} \times BD \times AC = \frac{1}{2} \times 24 \times 10 = 120$ cm^2.



20 A peace of land has the shape of a trapezium whose area is 4000 m^2 , the lengths of the two parallel bases and its height of ratio 3 : 2 : 4 , respectively , find the length of its middle base.

SOLUTION

Let the length of the two bases are 3 x , 2 x and its height 4 x

$\because$ The area of the trapezium $= \frac{1}{2}(3x + 2x) \times 4x$ $\therefore$ $4000 = 5x \times 2x$

$\therefore$ $4000 = 10x^2$ $\therefore$ $x^2 = 4000 \div 10 = 400$ $\therefore$ $x = 20$

$\therefore$ The two bases are 60 cm. and 40 cm.

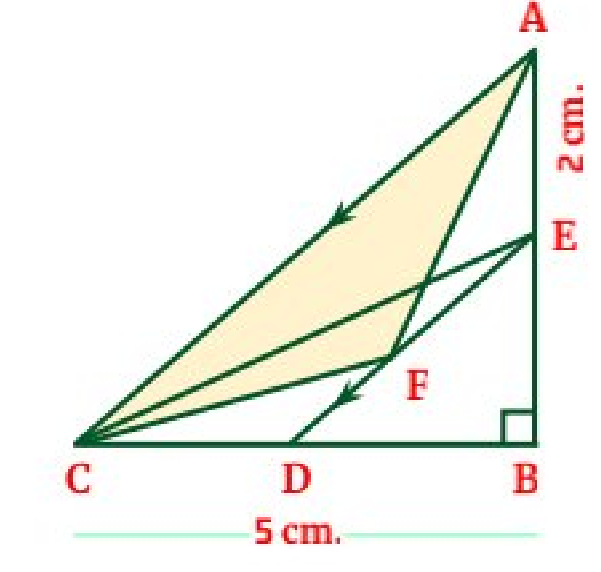
$\therefore$ The length of the middle base $= \frac{1}{2}(60 + 40) = 50$ cm.

21 In the opposite figure :

ABC is a right-angled triangle at B in which : BC = 5 cm. , E ∈ $\overline{AB}$, D ∈ $\overline{BC}$

Where $\overline{ED}$ // $\overline{AC}$ and AE = 2 cm.

Find : The area of $\triangle$ AFC.



SOLUTION

In $\triangle$ ACE

$\because$ $\overline{CB} \perp \overrightarrow{AE}$ $\therefore$ its area $= \frac{1}{2} \times AE \times CB = \frac{1}{2} \times 2 \times 5 = 5$ cm^2.

In $\triangle$ AEC and $\triangle$ AFC { $\overline{AC}$ is a common base , $\overline{AC}$ // $\overline{ED}$, F ∈ $\overline{ED}$ }

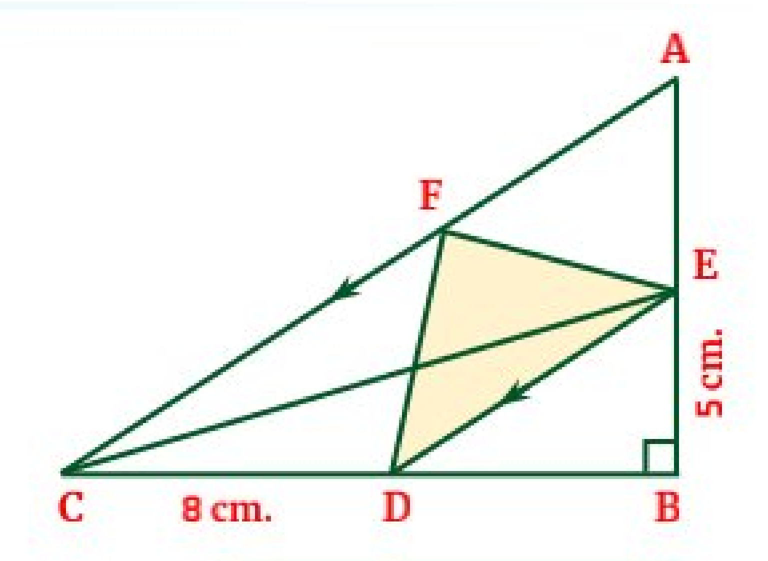
$\therefore$ The area of $\triangle$ DBE = the area of $\triangle$ DCE = 5 cm^2.

22 In the opposite figure :

EBC is a right-angled triangle at B in which : EB = 5 cm. , D ∈ $\overline{CB}$, E ∈ $\overline{AB}$

Where $\overline{ED}$ // $\overline{AC}$ and CD = 5 cm.

Find : The area of $\triangle$ AFC.



SOLUTION

In $\triangle$ ECD

$\because$ $\overline{EB} \perp \overrightarrow{CD}$ $\therefore$ its area $= \frac{1}{2} \times CD \times EB = \frac{1}{2} \times 8 \times 5 = 20$ cm^2.

In $\triangle$ EFD and $\triangle$ ECD { $\overline{ED}$ is a common base , $\overline{AC}$ // $\overline{ED}$, F ∈ $\overline{AC}$ }

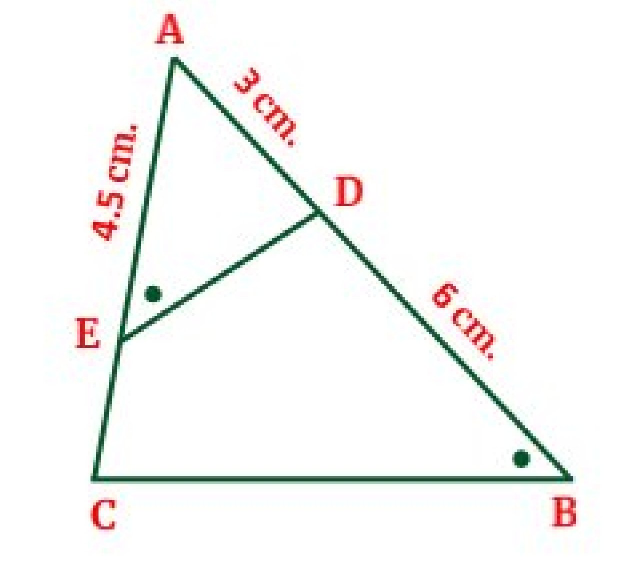
$\therefore$ The area of $\triangle$ EFD = the area of $\triangle$ ECD = 20 cm^2.

2 UNIT 5

Similarity – converse of phythagoras' theorem and edclidean theorem.



1 In the opposite figure :



m (∠ AED) = m (∠ B) , AD = 3 cm. , AE = 4.5 cm. and BD = 6 cm.

1 Prove that : △ ADE ~ △ ACB

2 Find : the length of $\overline{EC}$

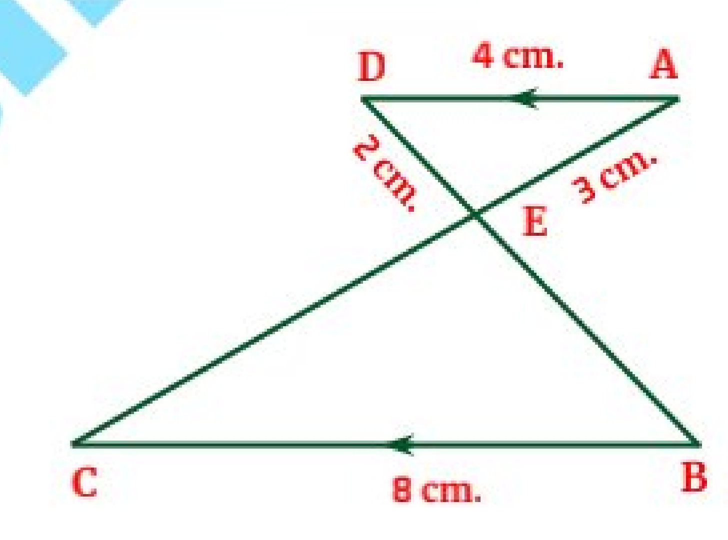
SOLUTION

In △ ABE and △ ACB { ∠ A is a common angle and m (∠ AED) = m (∠ B) }

∴ △ ADE ~ △ ACB ∴ $\frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$

∴ $\frac{3}{AC} = \frac{4.5}{9}$ ∴ AC $= \frac{3 \times 9}{4.5} = 6$ cm. ∴ EC = 6 – 4.5 = 1.5 cm.

2 In the opposite figure :



$\overline{AD}$ // $\overline{BC}$, AD = 4 cm. , AE = 3 cm. , DE = 2 cm. and BC = 8 cm.

1 Prove that : △ AED ~ △ CEB

2 Find : the perimeter of △ EBC

SOLUTION

∵ $\overline{AD}$ // $\overline{BC}$

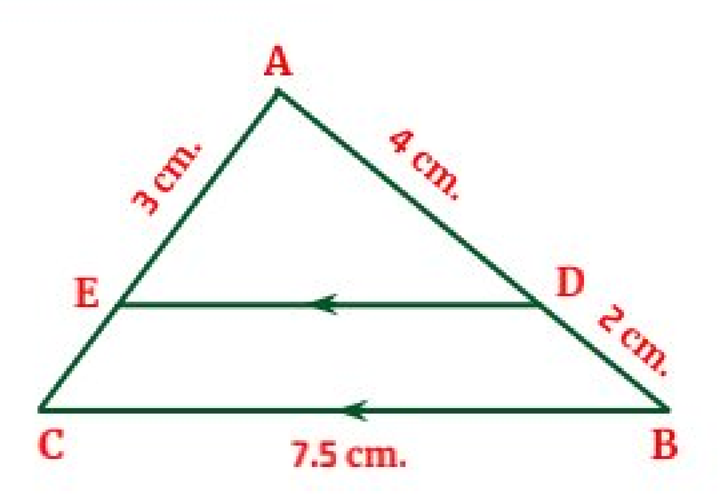
∴ m (∠ A) = m (∠ C) and m (∠ D) = m (∠ B) corresponding angles

∴ △ AED ~ △ CEB ∴ $\frac{AE}{CE} = \frac{DE}{EB} = \frac{AD}{CB}$

∴ $\frac{3}{AC} = \frac{2}{EB} = \frac{4}{8}$ ∴ AC $= \frac{3 \times 8}{4} = 6$ cm. and EB $= \frac{2 \times 8}{4} = 4$ cm.

∴ the perimeter of △ EBC = 8 + 6 + 4 = 18 cm.

3 In the opposite figure :



$\overline{DE}$ // $\overline{BC}$, AD = 4 cm. , AE = 3 cm. , BD = 2 cm. and BC = 7.5 cm.

1 Prove that : △ ADE ~ △ ABC

2 Find : the perimeter of △ ADE

SOLUTION

∵ $\overline{ED}$ // $\overline{BC}$

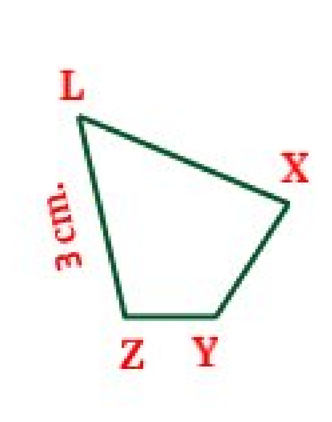
∴ m (∠ ADE) = m (∠ B) and m (∠ AED) = m (∠ C) corresponding angles

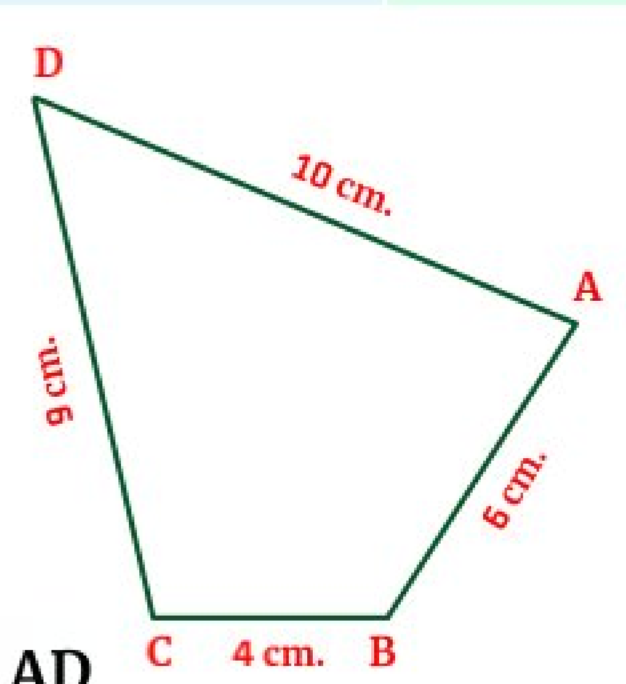
∴ △ ADE ~ △ ABC ∴ $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

∴ $\frac{4}{6} = \frac{DE}{7.5} = \frac{3}{AC}$ ∴ AC $= \frac{3 \times 6}{4} = 4.5$ cm. and ED $= \frac{4 \times 7.5}{6} = 5$ cm.

∴ the perimeter of △ ADE = 3 + 4 + 5 = 12 cm.

4 **In the opposite figure :**





The polygon ABCD ~ The polygon XYZL , AB = 6 cm.

, BC = 4 cm. , CD = 9 cm. , DA = 10 cm. and ZL = 3 cm.

Find : the perimeter of △ The polygon XYZL

SOLUTION

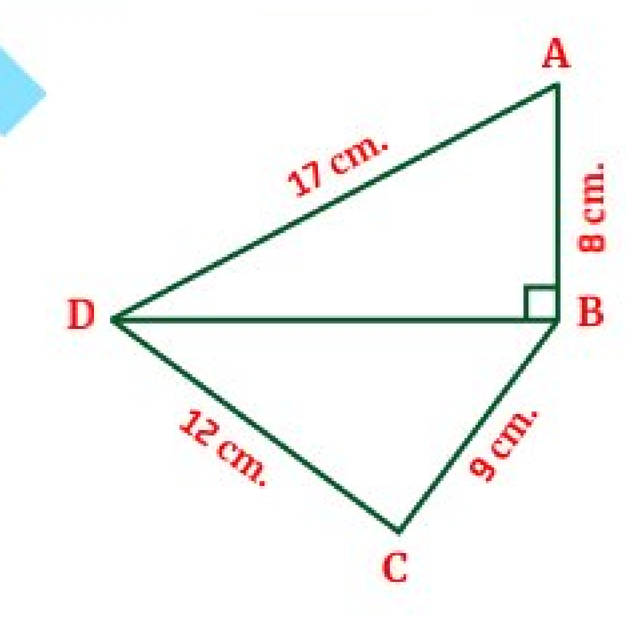
∴ The polygon ABCD ~ The polygon XYZL ∴ $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{AD}{XL}$

∴ $\frac{6}{XY} = \frac{4}{YZ} = \frac{9}{3} = \frac{10}{XL}$ ∴ $XY = \frac{3 \times 6}{9} = 2$ cm.

, $YZ = \frac{3 \times 4}{9} = 1\frac{1}{3}$ cm. , $XL = \frac{10 \times 3}{9} = 3\frac{1}{3}$ cm.

∴ the perimeter of △ The polygon XYZL $= 3 + 2 + 1\frac{1}{3} + 1\frac{1}{3} = 7\frac{2}{3}$ cm.

5 **In the opposite figure :**



ABCD is a quadrilateral in which AB = 8 cm. , BC = 9 cm. and CD = 12 cm.

AD = 17 cm. and $\overline{BD} \perp \overline{AB}$

[1] **Find :** the length of $\overline{BD}$

[2] **Prove that :** m (∠ C) = 90°

SOLUTION

∴ The △ ABD is a right-angled triangle at B

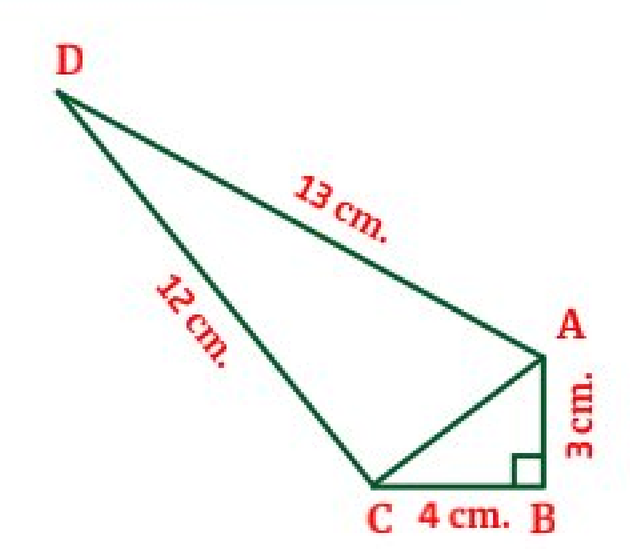
∴ $(BD)^2 = (AD)^2 - (AB)^2 = (17)^2 - (8)^2 = 289 - 64 = 225$ ∴ $BD = \sqrt{225} = 15$ cm.

In △ CBD

∵ $(BD)^2 = (15)^2 = 225$, $(CB)^2 + (CD)^2 = (9)^2 + (12)^2 = 81 + 144 = 225$

∴ $(BD)^2 = (CB)^2 + (CD)^2$ ∴ m (∠ C) = 90°

6 **In the opposite figure :**



BC = 4 cm. , AD = 13 cm. , AB = 3 cm.

, CD = 12 cm. and m (∠ B) = 90°

[1] **Find :** the length of $\overline{AC}$

[2] **Prove that :** m (∠ ACD) = 90°

SOLUTION

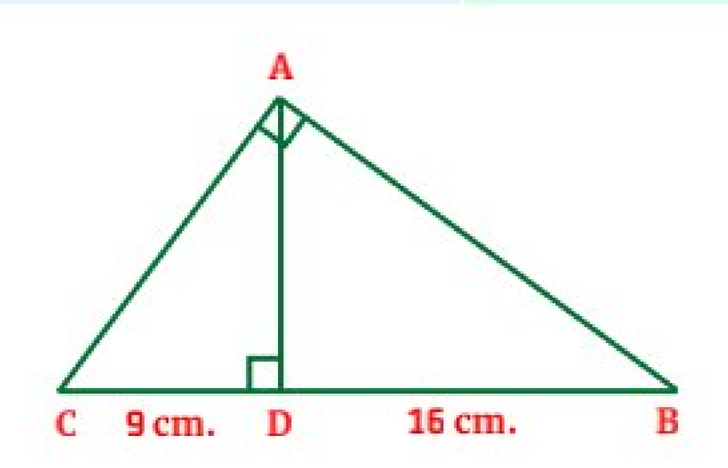
∵ The △ ABC is a right-angled triangle at B

∴ $(AC)^2 = (AB)^2 + (BC)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$ ∴ $AC = \sqrt{25} = 5$ cm.

In △ ACD : $(AD)^2 = (13)^2 = 169$, $(CD)^2 + (AC)^2 = (12)^2 + (5)^2 = 144 + 25 = 169$

∴ $(BD)^2 = (CB)^2 + (CD)^2$ ∴ m (∠ ACD) = 90°

7 **In the opposite figure :**



ABC is a right-angled triangle at A , $\overline{AD} \perp \overline{BC}$, CD = 9 cm. , CD = 16 cm.

Find : the length of $\overline{AC}$, $\overline{AD}$ and $\overline{AB}$

SOLUTION

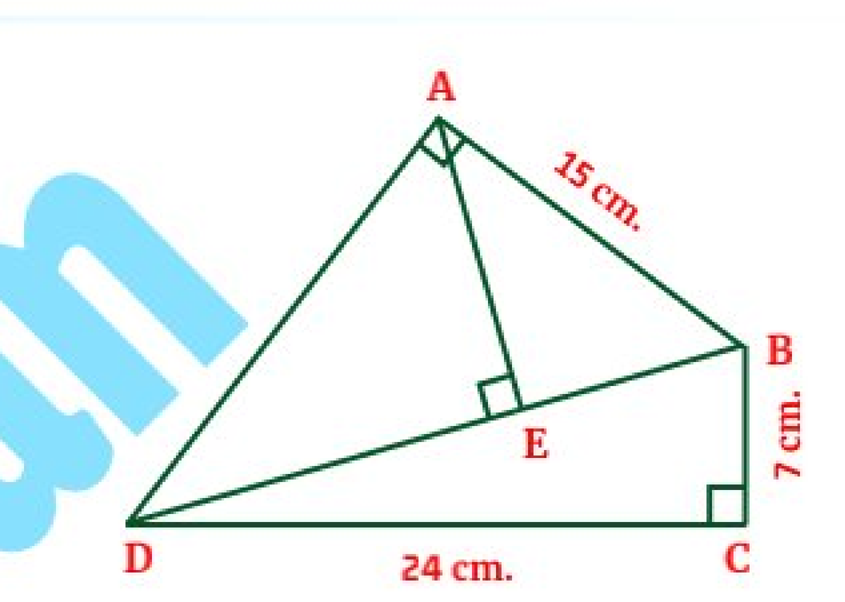
$\because$ The $\triangle$ ABC is a right-angled triangle at B , and $\overline{AD} \perp \overline{BC}$

$\therefore (AC)^2 = CD \times CB = 9 \times 25 = 225$ $\therefore AC = \sqrt{225} = 15$ cm.

$, (AD)^2 = CD \times DB = 9 \times 16 = 144$ $\therefore AC = \sqrt{144} = 12$ cm.

$, (AB)^2 = BD \times CB = 16 \times 25 = 400$ $\therefore AC = \sqrt{400} = 20$ cm.

8 **In the opposite figure :**



ABCD is a quadrilateral , where m ($\angle$ BCD) = m ($\angle$ BAD) = 90°

$\overline{AE} \perp \overline{BD}$, BC = 7 cm. , CD = 24 cm. and AB = 15 cm. **Find :**

1. The length of $\overline{BD}$ and $\overline{AD}$
2. The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$

SOLUTION

$\because$ The $\triangle$ BCD is a right-angled triangle at C

$\therefore (BD)^2 = (BC)^2 + (CD)^2 = (7)^2 + (24)^2 = 49 + 576 = 625$ $\therefore AC = \sqrt{625} = 25$ cm.

$\because$ The $\triangle$ ABD is a right-angled triangle at A

$\therefore (AD)^2 = (BD)^2 - (AB)^2 = (25)^2 - (15)^2 = 625 - 225 = 400$ $\therefore AC = \sqrt{400} = 20$ cm.

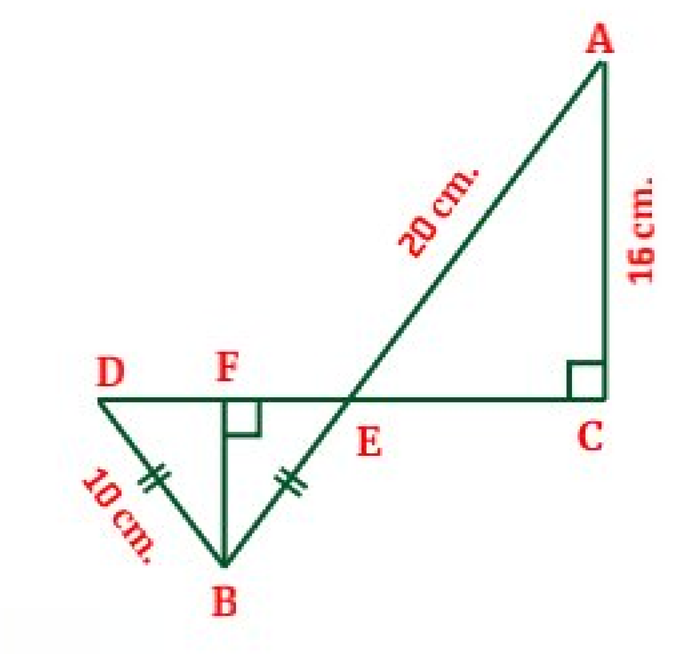
$\because \overline{AE} \perp \overline{BD}$ $\therefore$ the projection of A on $\overleftrightarrow{BD}$ is E

$\because B \in \overleftrightarrow{BD}$ $\therefore$ the projection of B on $\overleftrightarrow{BD}$ is B

$\therefore$ the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$ is $\overline{EB}$

$\because (AB)^2 = BE \times BD$ $\therefore 225 = 25\,BE$ $\therefore BE = 225 \div 25 = 9$ cm.

9 **In the opposite figure :**



$\overline{AB} \cap \overline{CD} = \{E\}$, E is the midpoint of $\overline{CD}$, AC = 16 cm.

, AE = 20 cm. and BD = BE = 10 cm.

Find : The length of the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$

SOLUTION

$\because$ AEC is a right-angled triangle at C.

$\therefore (EC)^2 = (AE)^2 - (AC)^2 = (20)^2 - (16)^2 = 400 - 256 = 144$ $\therefore EC = \sqrt{144} = 12$ cm.

$\because$ E is the midpoint of $\overline{CD}$ $\therefore EC = DE = 12$ cm.

In $\triangle$ BDE , BE = BD and $\overline{BE} \perp \overline{DE}$

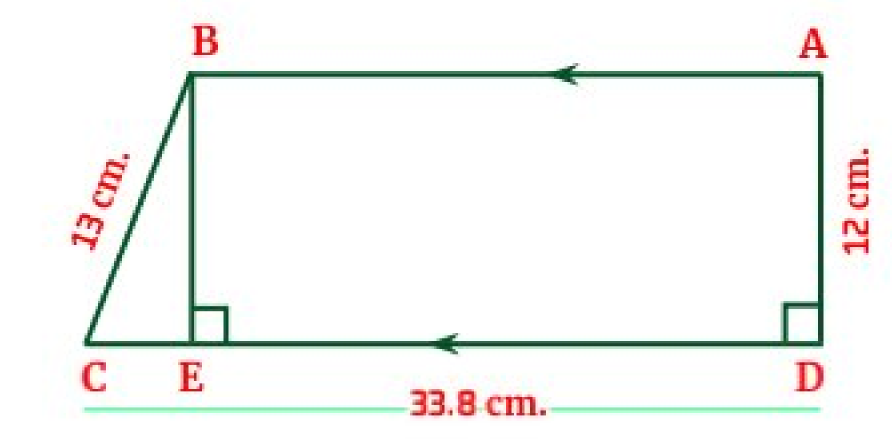
$\therefore$ F is the midpoint of $\overline{DE}$ $\therefore$ DF = FE = 6 cm.

$\because$ $\overline{AC} \perp \overline{CD}$ $\therefore$ the projection of A on $\overleftrightarrow{CD}$ is C

$\because$ $\overline{BF} \perp \overline{CD}$ $\therefore$ the projection of B on $\overleftrightarrow{CD}$ is F

$\therefore$ the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$ is $\overline{FC}$ $\therefore$ FC = 6 + 12 = 18 cm.

10 **In the opposite figure :**



ABCD is a trapezium in which $\overline{AB}$ // $\overline{DC}$, $\overline{AD} \perp \overline{DC}$, AD = 12 cm.

BC = 13 cm. , DC = 33.8 cm. , $\overline{BE}$ // $\overline{DC}$.

1 **Find :** The length of $\overline{CE}$, $\overline{AB}$ and $\overline{DB}$

2 **Find :** The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$

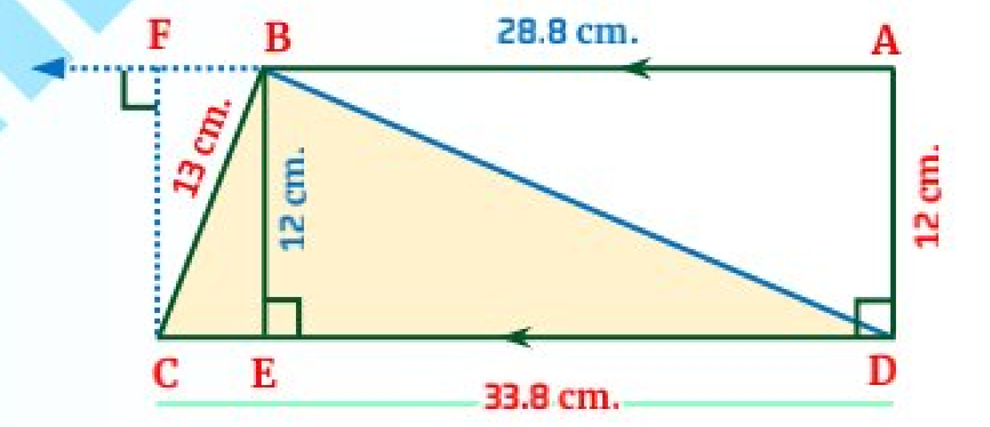
3 **Prove that :** m ($\angle$ DBC) = 90°

SOLUTION

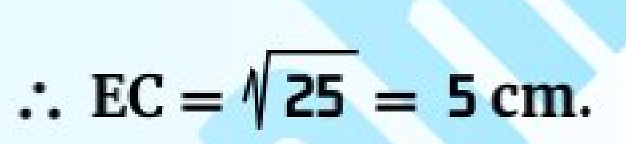
$\because$ $\overline{AB}$ // $\overline{DC}$, $\overline{AD} \perp \overline{DC}$ and $\overline{BE}$ // $\overline{DC}$ $\therefore$ the figure ABED is a rectangle

$\therefore$ AD = EB = 12 cm. and AB = ED

$\because$ BEC is a right–angled triangle at E.



$\therefore$ $(EC)^2$ = 169 – 144 = 25 $\therefore$ EC = $\sqrt{25}$ = 5 cm.



$\therefore$ AB = ED = 33.8 – 5 = 28.8 cm.

$\because$ ABD is a right–angled triangle at A.

$\therefore$ $(BD)^2 = (28.8)^2 + (12)^2 = 973.44$ $\therefore$ EC = $\sqrt{973.44}$ = 31.2 cm.

$\because$ $\overline{DA} \perp \overleftrightarrow{AB}$ $\therefore$ the projection of D on $\overleftrightarrow{CD}$ is A

$\because$ $\overline{CF} \perp \overleftrightarrow{AB}$ $\therefore$ the projection of C on $\overleftrightarrow{CD}$ is F

$\therefore$ the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$ is $\overline{AF}$ $\therefore$ AF = CD = 33.8 cm.

In $\triangle$ CBD : $(CD)^2 = (33.8)^2 = 1142.44$, $(BD)^2 + (BC)^2 = 973.44 + 169 = 1142.44$

$\therefore$ $(CD)^2 = (BD)^2 + (BC)^2$ $\therefore$ m ($\angle$ DBC) = 90°

11 $\triangle$ ABC where AB = 6 cm. , BC = 8 cm. , AC = 4 cm. , determine the type of the angle BAC.

SOLUTION

$\because$ $(BC)^2 = (8)^2 = 64$, $(AB)^2 + (AC)^2 = 36 + 16 = 52$ $\therefore$ $(BC)^2 > (AB)^2 + (AC)^2$

$\therefore$ $\angle$ BAC is an obtuse angle

12 determine the type of the angle C in $\triangle$ ABC in which AB = 7 cm. , BC = 3 cm. , AC = 5 cm.

SOLUTION

$\because$ $(AB)^2 = (7)^2 = 49$, $(BC)^2 + (AC)^2 = 9 + 25 = 34$ $\therefore$ $(AB)^2 > (BC)^2 + (AC)^2$

$\therefore$ $\angle$ BAC is an obtuse angle

13 determine the type of $\triangle$ ABC according to its angles if AB = 3.5 cm. , BC = 2.5 cm. , AC = 3 cm.

SOLUTION

$\because (AB)^2 = (3.5)^2 = 12.25$, $(BC)^2 + (AC)^2 = 6.25 + 9 = 15.25$ $\therefore (AB)^2 < (BC)^2 + (AC)^2$

$\therefore \angle$ BAC is an acute-angled triangle.

14 $\triangle ABC \sim \triangle EFD$, AB = 4 cm. , BC = 5 cm. , AC = 6 cm. , If the perimeter of $\triangle$ EFD = 60 cm.

Find : The length of sides of $\triangle$ EFD.

SOLUTION

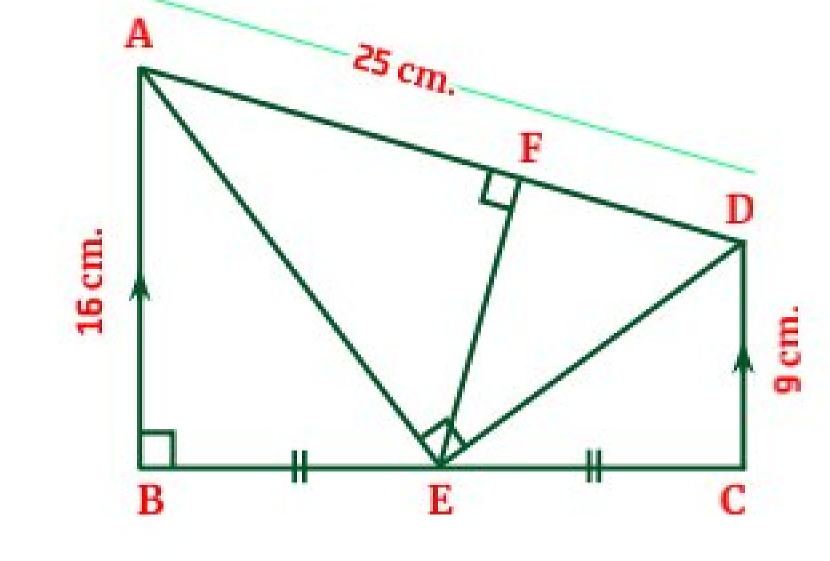
The perimeter of $\triangle$ ABC = 5 + 6 + 4 = 15 cm.

$\because \triangle ABC \sim \triangle EFD$ $\therefore \frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED} = \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle EFD}$

$\therefore \frac{4}{EF} = \frac{5}{FD} = \frac{6}{ED} = \frac{15}{60}$ $\therefore EF = \frac{4 \times 60}{15} = 16$ cm.

, $FD = \frac{5 \times 60}{15} = 20$ cm. and $ED = \frac{6 \times 60}{15} = 24$ cm.

15 **In the opposite figure :**



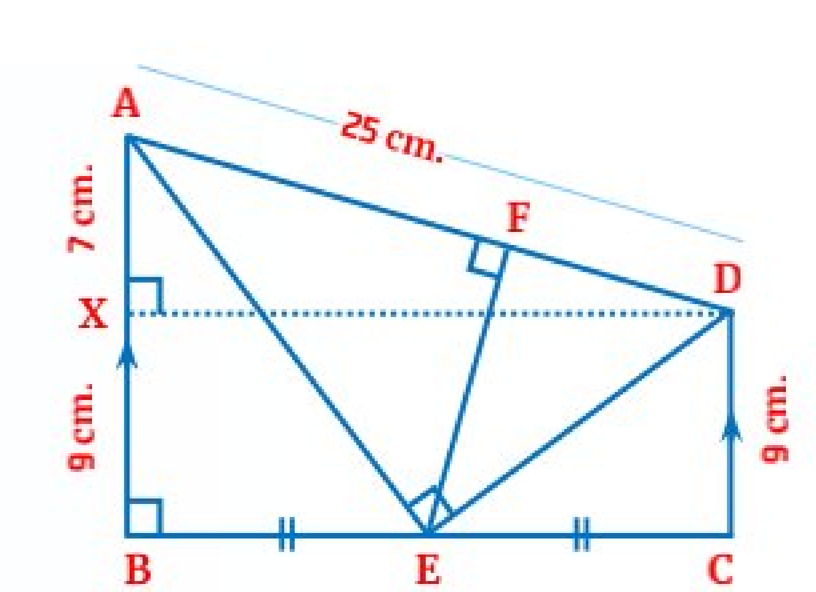
ABCD is a trapezium in which $\overline{AB} \,//\, \overline{DC}$, E is the midpoint of $\overline{BC}$

And m ($\angle$ ABC) = 90° , AB = 16 cm. , AD = 25 cm. and DC = 9 cm.

$\overline{AE} \perp \overline{ED}$, $\overline{EF} \perp \overline{AD}$, Find :

1. The area of the trapezium ABCD
2. The length of $\overline{EF}$

SOLUTION



Draw $\overline{DX} \perp \overline{AB}$

$\because \overline{XB} \,//\, \overline{DC}$, $\overline{DX} \perp \overline{AB}$ and $\overline{CB} \perp \overline{AB}$

$\therefore$ the figure XDCB is a rectangle

$\therefore$ DC = XB = 9 cm. and XD = BC , AX = 16 – 7 = 9 cm.

$\because$ AXD is a right-angled triangle at X.

$\therefore (XD)^2 = (AD)^2 - (AX)^2 = 625 - 49 = 576$ $\therefore XD = \sqrt{576} = 24$ cm.

$\therefore$ XD = BC = 24 cm. $\therefore$ the area $= \frac{1}{2}(16 + 9) \times 24 = 300$ cm^2

$\because$ ABE is a right-angled triangle at B.

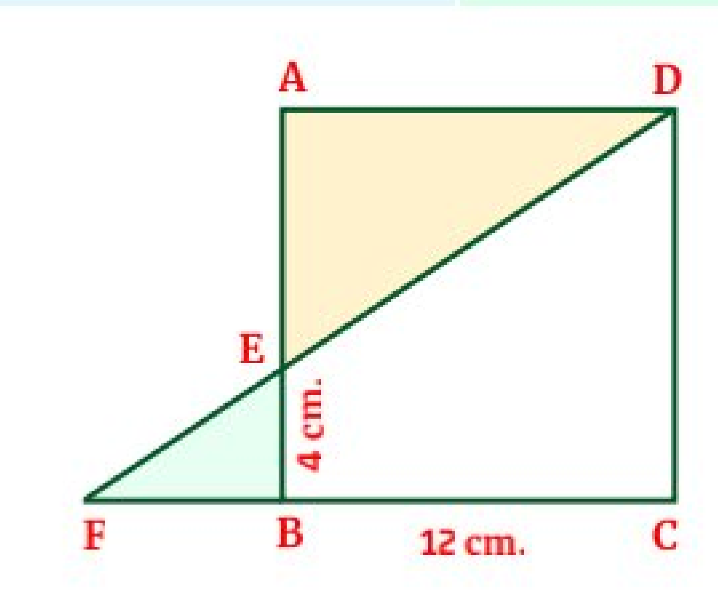
$\therefore (AE)^2 = (AB)^2 + (BE)^2 = 256 + 144 = 400$ $\therefore AE = \sqrt{400} = 20$ cm.

$\because$ DEC is a right-angled triangle at C.

$\therefore (DE)^2 = (DC)^2 + (CE)^2 = 81 + 144 = 225$ $\therefore DE = \sqrt{225} = 15$ cm.

$\therefore EF = \frac{AE \times ED}{AD} = \frac{15 \times 20}{25} = 12$ cm.

16 **In the opposite figure :**



ABCD is a square of side length 12 cm. , $B \in \overrightarrow{CB}$

where $\overline{AB} \cap \overline{DF} = \{ E \}$ and EB = 4 cm.

1 **Prove that :** $\triangle$ ADE $\sim$ $\triangle$ BFE

2 **Find :** The length of $\overline{FB}$

SOLUTION

$\because$ ABCD is a square $\therefore$ AB = BC = 12 cm. , m ($\angle$ B) = m ($\angle$ C) = 90°

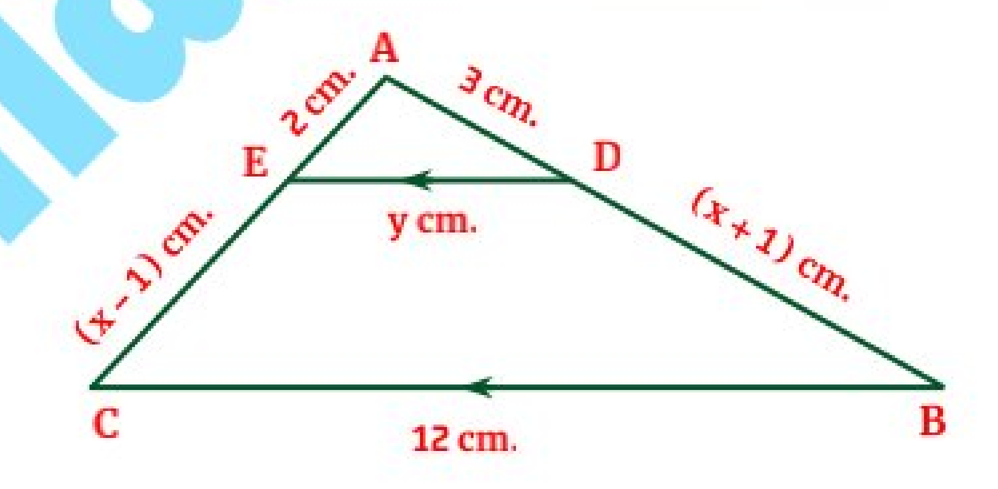
$\therefore$ AE = 12 – 4 = 8 cm.

In $\triangle\triangle$ ADE and BFE { m ($\angle$ B) = m ($\angle$ C) = 90° and m ($\angle$ AED) = m ($\angle$ FEB) (V.O.A) }

$\therefore$ $\triangle$ ADE $\sim$ $\triangle$ BFE $\therefore \frac{AD}{BF} = \frac{DE}{FE} = \frac{AE}{BE}$

$\therefore \frac{12}{BF} = \frac{8}{4}$ $\therefore BF = \frac{4 \times 12}{8} = 6$ cm.

17 **In the opposite figure :**



ABC is a triangle in which : $D \in \overline{AB}$ and $E \in \overline{AC}$ such that :

$\overline{DE} \parallel \overline{BC}$, AD = 3 cm. , AE = 2 cm. , BC = 12 cm. , DE = y cm.

BD = (x + 1) cm. and EC = (x – 1) cm.

And m ($\angle$ ABC) = 90° , AB = 16 cm. , AD = 25 cm. and DC = 9 cm.

$\overline{AE} \perp \overline{ED}$, $\overline{EF} \perp \overline{AD}$, **Find :** The length of $\overline{AB}$, $\overline{EC}$ and $\overline{DE}$

SOLUTION

AC = 2 + x – 1 = (x + 1) cm. and AB = x + 1 + 3 = (x + 4) cm.

$\therefore$ m ($\angle$ ADE) = m ($\angle$ B) and m ($\angle$ AED) = m ($\angle$ C) corresponding angles

$\therefore$ $\triangle$ ADE $\sim$ $\triangle$ ABC $\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

$\therefore \frac{3}{x+4} = \frac{y}{12} = \frac{2}{x+1}$ $\therefore \frac{3}{x+4} = \frac{2}{x+1}$ $\therefore 3x + 3 = 2x + 8$

$\therefore$ x = 8 – 3 = 5 cm.

, $\frac{3}{9} = \frac{y}{12}$ $\therefore y = \frac{3 \times 12}{9} = 4$ cm.

$\therefore$ AC = 5 + 1 = 6 cm , AB = 5 + 4 = 9 cm. , and DE = 4 cm.

Best wishes, Mr. Abdelrahman Essam



Choose the correct answer :

1.	In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then $\angle$ C is …………… (a) acute. (b) right. (c) obtuse. (d) straight.
2.	A rhombus whose diagonals lengths are 6 cm. , 10 cm. has area …………… cm^2 (a) 60 (b) 30 (c) 15 (d) 10
3.	The ratio between the lengths of two corresponding sides of two similar polygons is 3 : 5 , then the ratio between their perimeters is …………… (a) 2 : 5 (b) 5 : 3 (c) 3 : 5 (d) 1 : 2
4.	A square of perimeter 20 cm. , then its area equals …………… cm^2 (a) 20 (b) 25 (c) 50 (d) 100

5. If the area of a trapezium is 100 cm^2 and its height is 5 cm. , then the length of its middle base = …………… cm.

(a) 20 (b) 30 (c) 40 (d) 50

6. The median of a triangle divides its surface into two triangles ……………

(a) congruent. (b) equal in area. (c) similar. (d) coincide.

7. A trapezium whose bases lengths are 6 cm. , 8 cm. , then the length of its middle base equals …………… cm.

(a) 48 (b) 24 (c) 14 (d) 7

8. If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is …………… cm.

(a) 30 (b) 45 (c) 60 (d) 75

9. If the area of the triangle is 24 $cm.^2$ and its height = 8 cm. , then the length of the corresponding base …………… cm.

(a) 16 (b) 6 (c) 3 (d) 12

10. ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, then the projection of $\overline{BD}$ on $\overleftrightarrow{AC}$ is ……………

(a) A (b) B (c) C (d) D

11. The area of parallelogram whose length of its base 6 cm. and its corresponding height of this base 4 cm. equals ……… cm^2.

(a) 12 (b) 20 (c) 24 (d) 48

12. The triangle whose lengths of its sides 6 cm. , 8 cm. , 10 cm. is ………

(a) acute-angled triangle. (b) right-angled triangle.

(c) obtuse-angled triangle. (d) otherwise.

13. The rhombus whose lengths of its diagonals 6 cm. and 10 cm. , then its area = ……… cm^2.

(a) 60 (b) 30 (c) 15 (d) 10

14. Trapezium of length of its middle base 8 cm. and surface area 56 cm^2 , then its height = ... cm.

(a) 32 (b) 24 (c) 448 (d) 7

15. All .... are similar.

(a) squares (b) triangles (c) rectangles (d) parallelograms

16. A square of diagonal length 12 cm. , then its area = ... cm^2

(a) 24 (b) 36 (c) 48 (d) 72

17. In ΔABC if $(AC)^2 = (AB)^2 + (BC)^2$, then $\angle$ .... is right.

(a) A (b) B (c) C (d) otherwise

18. ABC is a triangle where AB = 2 cm. , BC = 6 cm. and CA = 5 cm. , then $m(\angle A)$ .... $90°$

(a) < (b) > (c) = (d) ≥

19. If $\Delta ABC \sim \Delta XYZ$, $m(\angle B) = 50°$, then $m(\angle Y) =$ ....

(a) 30° (b) 40° (c) 50° (d) 60°

20. If the ratio between the length of two corresponding sides in two similar triangles is equal to 1 , then the two triangles are ....

(a) congruent. (b) different. (c) parallel. (d) otherwise.

21. * If the lengths of two adjacent sides of a parallelogram are 8 cm. and 10 cm. and its greater height is 5 cm. , then its area = ... cm^2

(a) 80 (b) 50 (c) 40 (d) 18

22. The length of the two adjacent sides in a parallelogram are 7 cm. , 5 cm. and the length of its smallest height is 4 cm. , then the area of the parallelogram equals .... cm^2

(a) 35 (b) 25 (c) 28 (d) 49

23. If the ratio of enlargement between two similar triangles equals .... , then the two triangles are congruent.

(a) 1 (b) 2 (c) 0.5 (d) 0.25

24. If Δ ABC in which $(AB)^2 + (BC)^2 < (AC)^2$, then ($\angle$ B) is ....

(a) acute. (b) right. (c) reflex. (d) obtuse.

---

25. If the projection of a line segment on a straight line is a point , then the line segment .... the straight line.

(a) // (b) $\perp$ (c) $\equiv$ (d) $\subset$

---

26. If $\Delta ABC \sim \Delta DEO$, $AB = \frac{1}{3}$ DE , then the perimeter of Δ ABC equals .... the perimeter of Δ DEO

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 3 (d) 9

---

27. ✱ The ratio between the area of the parallelogram and the area of the triangle whose base is common and are included between two parallel straight lines = ...

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 2 : 3

---

28. The length of the base of a triangle whose area 36 cm^2 and height 8 cm. is .... cm.

(a) 6 (b) 9 (c) 18 (d) 20

---

29. If $\overline{AB} \parallel \overleftrightarrow{XY}$, then the length of the projection of $\overline{AB}$ on $\overleftrightarrow{XY}$ .... length of $\overline{AB}$

(a) $<$ (b) $>$ (c) $=$ (d) $\geq$

---

30. The area of the trapezium whose middle bases 7 cm. , and height 6 cm. = ... cm^2

(a) 21 (b) 42 (c) 40 (d) 13

---

31. If the area of a parallelogram is 80 cm^2 and one of its bases length 10 cm. , then the length of the corresponding height of this base = ... cm.

(a) 8 (b) 6 (c) 7 (d) 20

---

32. Δ ABC in which AB = 4 cm. , BC = 6 cm. , AC = 8 cm. , then m ($\angle$ B) .... 90°

(a) $>$ (b) $<$ (c) $=$ (d) twice

---

33. ✱ The length of the base of a triangle whose area 30 cm^2 and height 6 cm. is .... cm.

(a) 5 (b) 10 (c) 15 (d) 20

---

34. In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then angle C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

35. If $\overline{AB} \parallel \overline{XY}$, then the length of the projection of $\overline{AB}$ on $\overline{XY}$ .... the length of $\overline{AB}$

(a) > (b) ≤ (c) = (d) <

36. A rhombus whose diagonal lengths 12 cm. , 9 cm. , then its area = ... cm^2.

(a) 18 (b) 54 (c) 45 (d) 108

37. Area of the trapezium whose base lengths are 6 cm. , 8 cm. and its height 10 cm. = ... cm^2.

(a) 140 (b) 480 (c) 70 (d) 120

38. ABC is a triangle in which $(AB)^2 = (BC)^2 + (AC)^2$ and m (∠ B) = 40° , then m (∠ A) = ...

(a) 40° (b) 50° (c) 90° (d) 130°

39. ✱ The median of a triangle divides its surface into two ....

(a) congruent triangles. (b) triangles equal in area.
(c) isosceles triangle. (d) right-angled triangle.

40. **In the opposite figure :**

EY × EZ = ...

(a) $(XY)^2$ (b) $(XZ)^2$
(c) $(XE)^2$ (d) $(YZ)^2$

Complete each of the following :

1. Two triangles which have the same base and the vertices opposite to this base lie on a straight line parallel to the base have ....

2. In Δ ABC , If $(AC)^2 + (BC)^2 = (AB)^2$, then m (∠ ....) = 90°

3. If the point A ∈ the line L , then the projection of the point A on the line L is ....

4. If the area of a parallelogram is 35 $cm.^2$ and one of its bases length is 10 cm. , then the length of the corresponding height of this base = ... cm.

5. A trapezium whose bases lengths are 8 cm. , 10 cm. and its height is 5 cm. , then its area equals .... $cm.^2$

6. The two polygons are similar if their corresponding sides are .... and their corresponding angles are ....

7.	The area of a rhombus is 24 $cm.^2$, the length of one of its diagonals is 8 cm. , then the length of the other diagonal is ....
8.	In Δ ABC , if $(AB)^2 = (AC)^2 - (BC)^2$, then Δ ABC is right-angled at ....
9.	A triangle whose side lengths are 6 cm. , 8 cm. and 11 cm. , then its type according to its angles is ....
10.	Area of triangle is equal to half of area of a parallelogram if they have a common ....
11.	The projection of point on a straight line is ....
12.	If the triangle ABC is obtuse-angled triangle at B , then $(AC)^2$ .... $(AB)^2 + (BC)^2$
13.	The square whose length of its diagonal 8 cm. , then its area = ... cm^2.
14.	The two triangles have same base and the vertices opposite to this base on straight line parallel to the base ....
15.	Area of triangle = $\frac{1}{2}$ × .... × corresponding height.
16.	In Δ ABC , if $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle$ B is ....
17.	The two triangles are similar if the corresponding angles are .... **From the opposite figure :** (a) The projection of $\overline{CD}$ on $\overleftrightarrow{AB}$ is .... (b) The projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is .... C A D B
18.	A rhombus whose diagonal lengths are 6 cm. , 10 cm. has area .... cm^2.
19.	If $\Delta ABC \sim \Delta XYZ$, m ($\angle$ A) + m ($\angle$ B) = 60° , then m ($\angle$ Z) = ...
20.	The area of the trapezium whose parallel bases 6 cm. , 10 cm. and height 5 cm. equals ....
21.	The two polygons are similar to a third are ....
22.	The area of rhombus whose perimeter is 20 cm. and height 4 cm. = ...
23.	The projection of a point which belong to a straight line on this line is ....
24.	The area of the rhombus whose diagonals 6 cm. , 8 cm. equals .... cm^2.

25.	The two polygons are similar if the corresponding sides .... and their corresponding angles ....
26.	The diagonal of a square whose area 50 cm^2 equals .... cm.
27.	If two polygons are similar and the ratio between the lengths of two corresponding side is 1 : 3 and the perimeter of smaller polygons is 12 cm. , then the perimeter of the greater polygon is ....
28.	The ratio between the length of two corresponding sides of two similar polygon is 3 : 5 , then the ratio between their perimeter = ...
29.	If $\overline{AD} \perp \overline{BC}$, then the projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is ....
30.	A square of diagonal length 12 cm. , then its area = ... cm^2
31.	A triangle whose side lengths 6 cm. , 8 cm. , 11 cm. , then its type according to its angle is ....
32.	If $\Delta ABC \sim \Delta DEF$ and m ($\angle$ B) + m ($\angle$ C) = 70° , then m ($\angle$ D) = ...°

Essay problems:

1.	The sides lengths of one of two similar triangles are 3 cm. , 4 cm. , 5 cm. and the perimeter of the other triangle is 36 cm. find the side lengths of the other triangle.
2.	**In the opposite figure :** ABCD is a parallelogram , X $\in \overline{AB}$, Y $\in \overline{AD}$ such that : area of Δ CBX = area of Δ CYD **Prove that :** $\overline{XY} \parallel \overline{BD}$ D Y A X C B
3.	**In the opposite figure :** ABC is a triangle in which : BD = 2 cm. , CD = 8 cm. , AD = 4 cm. , $\overline{AD} \perp \overline{BC}$ **Prove that :** m ($\angle$ BAC) = 90° A 4cm. C 8cm. D 2cm. B
4.	ABCD is a parallelogram in which : AB = 18 cm. and BC = 12 cm. We draw $\overline{DE} \perp \overline{BC}$, $\overline{DO} \perp \overline{AB}$, DE = 15 cm. Calculate the area of parallelogram ABCD and find the length of $\overline{DO}$

5.	**In the opposite figure :** DEO is a right-angled triangle at E , $\overline{EN} \perp \overline{DO}$, DN = 16 cm. and ON = 9 cm. **Find the length of each of :** $\overline{EN}$, $\overline{ED}$, $\overline{EO}$
6.	**In the opposite figure :** $\overline{AO} \perp \overleftrightarrow{CB}$, $\overline{BE} \perp \overline{AC}$, AC = 10 cm. , BC = 7 cm. and AO = 5 cm. **Find :** (1) The length of $\overline{BE}$ (2) The area of Δ ABC
7.	ABCD is a parallelogram in which : AB = 8 cm. , AC = 20 cm. and BD = 12 cm. **Prove that :** m ($\angle$ ABD) = 90° , then find the area of this parallelogram.
8.	**In the opposite figure :** Δ DBA is a similar to Δ ABC , m ($\angle$ BAC) = 90° **Prove that :** $\overline{AD} \perp \overline{BC}$ and if AB = 8 cm. , AC = 6 cm. **Find the length of :** $\overline{BD}$
9.	**In the opposite figure :** ABC is a triangle , $\overline{AD} \perp \overline{BC}$ If AD = 24 cm. , AB = 26 cm. and AC = 30 cm. **Find :** BC , then calculate area of Δ ABC
10.	Δ EFD ~ Δ ABC , AB = 4 cm. , BC = 5 cm. , AC = 6 cm. , if the perimeter of Δ EFD = 60 cm. , find the length of sides Δ EFD
11.	**In the opposite figure :** Δ ABC ~ Δ AED , m ($\angle$ AED) = 44° , AD = 3 cm. , EA = 4 cm. , DB = 5 cm. , BC = 8 cm. **find the length of each of :** $\overline{ED}$ and $\overline{EC}$

12. **In the opposite figure :**

m (∠ AED) = m (∠ B) , AD = 3 cm.

, AE = 4.5 cm. and BD = 6 cm.

(1) **Prove that :** Δ ADE ~ Δ ACB

(2) **Find :** The length of $\overline{EC}$

13. **✱ In the opposite figure :**

ABC is a triangle with a median $\overline{AD}$

, E ∈ $\overline{AD}$, draw $\overline{BE}$ and $\overline{CE}$

Prove that : The area of Δ ABE = the area of Δ ACE

14. **In the opposite figure :**

ABCD is a quadrilateral in which

AB = 8 cm. , BC = 9 cm. and CD = 12 cm.

, AD = 17 cm. and $\overline{DB}$ ⊥ $\overline{AB}$

(1) **Find :** The length of $\overline{BD}$

(2) **Prove that :** m (∠ C) = 90°

15. If the lengths of the two parallel bases of a trapezium are 5 cm. , 7 cm. and if the length of its height 4 cm. , find its area

16. **In the opposite figure :**

ABCD is a quadrilateral , where m (∠ BCD) = m (∠ BAD) = 90°

, $\overline{AE}$ ⊥ $\overline{BD}$, BC = 7 cm. , CD = 24 cm. and AB = 15 cm.

Find : (1) The length of $\overline{BD}$ and $\overline{AD}$

(2) The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$

17. The ratio between the length of corresponding sides of two similar triangle is 3 : 5 and if the perimeter of the greater is 60 cm. , find the perimeter of the smaller triangles.

18. **In the opposite figure :**

AB = 3 cm. , BC = 4 cm. , AD = 13 cm.

, CD = 12 cm. , m (∠ B) = 90°

(1) **Find :** The length of : $\overline{AC}$

(2) **Prove that :** m (∠ ACD) = 90°

19. Δ ABC where AB = 6 cm. , BC = 8 cm. , AC = 4 cm. , determine the type of the angle BAC

20. **In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$, AD = 4 cm. , AE = 3 cm.

, DE = 2 cm. , BC = 8 cm.

(1) **Prove that :** Δ AED ~ Δ CED

(2) **Find :** The perimeter of Δ EBC

21. **In the opposite figure :**

m ($\angle$ ABC) = 90° , $\overline{BD} \perp \overline{AC}$

, AD = 1.8 cm. , DC = 3.2 cm.

Find : The length of each : $\overline{BD}$, $\overline{AB}$

22. **✱ In the opposite figure :**

ABC is a triangle in which

D $\in \overline{AB}$ and E $\in \overline{AC}$

such that the area of Δ ABE = the area of Δ ACD

Prove that : $\overline{DE} \parallel \overline{BC}$

23. **In the opposite figure :**

If Δ AXY ~ Δ ABC , AX = 7 cm. , AY = 6 cm. , YB = 8 cm.

(1) **Find :** The length of $\overline{XC}$

(2) **Find :** $\frac{XY}{BC}$

24. The length of the middle base of a trapezium is 30 cm. and the ratio between the length of its two parallel bases is 2 : 3

Find the length of each of them and if its height = 24 cm. , find its area.

25. **✱ In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$ and $\overline{AC} \cap \overline{BD} = \{M\}$

, D is the midpoint of $\overline{EC}$

Prove that : The area of Δ MDE = the area of Δ AMB

26.	**In the opposite figure :** In Δ ABC , BD = 9 cm. , DC = 16 cm. **Find :** Lengths of each of : $\overline{AD}$, $\overline{AB}$, $\overline{AC}$ A, B 9cm. D 16cm. C
27.	**In the opposite figure :** ABCD is a quadrilateral in which m ($\angle$ C) = 90° AB = AD = 13 cm. , BC = 6 cm. , CD = 8 cm. , E is midpoint of $\overline{BD}$ **Find :** The area of the shape ABCD A, 13cm., B, E, 6cm., D 8cm. C
28.	ABCD is trapezium in which $\overline{AD}$ // $\overline{BC}$, if BC = 2 AD = 20 cm. and its area = 180 cm^2, find the length of its height.
29.	**In the opposite figure :** AB = 3 cm. , BC = 4 cm. , AD = 13 cm. , CD = 12 cm. and m ($\angle$ ABC) = 90° **Prove that :** m ($\angle$ ACD) = 90° D, 13cm., 12cm., A, 3cm., C 4cm. B



Choose the correct answer :

1. Δ ABC in which AB = 3 cm. , BC = 6 cm. , and AC = 4 cm. , then m ($\angle$ B) .... 90°

   (a) < (b) > (c) = (d) ≤

2. If $\overline{AC}$ is the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$, then AC .... AB

   (a) < (b) > (c) = (d) ≤

3. If Δ ABC ~ Δ DEF and AB = $\frac{2}{5}$ DE , then the perimeter of Δ ABC = ... the perimeter of Δ DEF

   (a) 2 (b) 5 (c) $\frac{2}{5}$ (d) $\frac{4}{25}$

4. ABC is a right-angled triangle at B , AC = 10 cm. , BC = 8 cm. , then AB = ... cm.

   (a) 8 (b) 10 (c) 6 (d) 4

5. ABC is a triangle in which $(AB)^2 = (AC)^2 + (BC)^2$, m ($\angle$ B) = 40° , then m ($\angle$ A) = ...

   (a) 90° (b) 40° (c) 130° (d) 50°

6. ✱ The triangle whose base length is 6 cm. and its area is 24 cm^2 , the corresponding height = ... cm.

   (a) 4 (b) 8 (c) 3 (d) 18

7. A square of diagonal length 12 cm. , then its area = ... cm^2

   (a) 24 (b) 36 (c) 48 (d) 72

8. If Δ ABC ~ Δ DEF and m ($\angle$ B) + m ($\angle$ C) = 70° , then m ($\angle$ D) = ...

   (a) 70° (b) 35° (c) 140° (d) 110°

9.	The middle base of a trapezium = 12 cm. long and its height = 6 cm. , then its area = ... cm^2. (a) 72 (b) 36 (c) 9 (d) 18
10.	The length of the projection of a line segment on a given straight line .... the length of the line segment itself. (a) < (b) ≤ (c) ≥ (d) =
11.	ABC is an obtuse-angled triangle at A in which AB = 5 cm. , BC = 8 cm. , then AC = ... cm. (a) 5 (b) 7 (c) 8 (d) 13
12.	✱ The two triangles drawn on a common base their vertices located on a straight line parallel to the base are .... (a) congruent. (b) similar. (c) equal in perimeter. (d) equal in area.
13.	If the ratio of enlargement between two triangles equals 1 , then the two triangles are .... (a) congruent. (b) different. (c) right-angled. (d) coincide.
14.	The area of square of diagonal length 6 cm. is .... cm^2. (a) 18 (b) 36 (c) 12 (d) 6
15.	In ΔABC if $(AC)^2 + (AB)^2 < (BC)^2$, then $\angle A$ is .... (a) acute. (b) right. (c) obtuse. (d) straight.
16.	A trapezium whose middle base length is 8 cm. , then the length of the two parallel bases may be .... cm. (a) 3 , 5 (b) 6 , 10 (c) 4 , 6 (d) 4 , 4

Complete each of the following :

1.	The two polygons that are similar to third are ....
2.	The two diagonals of the isosceles trapezium are ....
3.	The two triangles are similar if its corresponding side lengths are ....
4.	The area of the trapezium = ... × ....

5. **Complete : In the opposite figure :**

ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$

(1) The projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ is ....

(2) $(BD)^2 = AD \times$ .... (3) $(BC)^2 = CA \times$ ....

(4) Δ ABC ~ Δ .... ~ Δ ....

(5) The perimeter of Δ ABC : the perimeter of Δ DBC = ...

6. 
The area of the square = $\frac{1}{2}$ ....

7. In Δ ABC , if $(AB)^2 = (BC)^2 + (AC)^2$, then m ($\angle$ ....) = 90°

8. The area of rhombus is 20 cm^2 , the length of one of its diagonals is 5 cm.
, then the length of the other diagonal = ...

9. If Δ ABC is right-angled at A and $\overline{AD} \perp \overline{BC}$, then $(AB)^2$ = ... × ....

10. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2$, then m ($\angle$ ....) = 90°

11. If the point A $\in$ the line L , then the projection of the point A on the line L is ....

12. A trapezium whose bases lengths are 8 cm. , 10 cm. , and its height is 5 cm.
, then its area equals .... cm^2.

13. The area of rhombus is 24 cm^2 , the length of one of its diagonals is 8 cm.
, then the length of other diagonal is ....

14. The two polygons that are similar to third are ....

Essay problems:

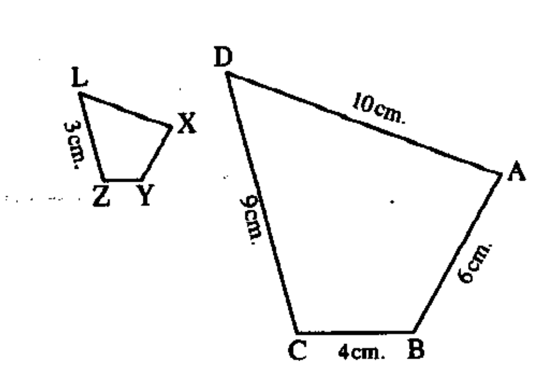
1. **In the opposite figure :**

The polygon ABCD ~ the polygon XYZL

, AB = 6 cm. , BC = 4 cm. , CD = 9 cm.

, DA = 10 cm. , ZL = 3 cm.

Find : The perimeter of the polygon XYZL



2. Determine the type of Δ ABC according to it's angles if AB = 3.5 cm. , BC = 2.5 cm. and AC = 3 cm.

3. **In the opposite figure :**
Δ ABC is right-angled at B , $\overline{BD} \perp \overline{AC}$
, AD = 9 cm. , and CD = 16 cm.
Find : (1) The length of $\overline{AB}$ (2) The length of $\overline{BD}$

4. **In the opposite figure :**
BC = 4 cm. , AD = 13 cm. , AB = 3 cm.
, DC = 12 cm. , m ($\angle$ B) = 90°
(1) **Find :** The length of $\overline{AC}$
(2) **Prove that :** m ($\angle$ ACD) = 90°

5. **✱ In the opposite figure :**
ABCD is a quadrilateral whose diagonals intersect at M
, $\overline{AD}$ // $\overline{BC}$, X $\in \overline{AD}$ and Y $\in \overline{AD}$
Such that : AX = DY
Prove that : The area of the figure ABMX = the area of the figure DCMY

6. Two similar polygons in which the ration between the lengths of two corresponding sides is 1 : 3 if the perimeter of the smaller is 20 cm. , find the perimeter of the greater.

7. **In the opposite figure :**
ABC is a triangle in which $\overline{DE}$ // $\overline{BC}$, BD = 2 cm.
, AD = 8 cm. , AE = 5 cm. , CE = x cm.
(1) **Prove that :** Δ ADE ~ Δ ABC
(2) **Find the value of :** x

8. **In the opposite figure :**
If Δ ABC ~ Δ ADE , AE = 6 cm.
, AD = 7 cm. and BE = 8 cm.
Find : (1) DC (2) $\frac{DE}{BC}$

9. **✱ In the opposite figure :**
$\overline{AD}$ // $\overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$ and BX = CY
Prove that :
The area of the figure ABXM = the area of the figure DCYM

(11) $(\sqrt{3})^{-4-3+9} \times (\sqrt{2})^{-5+7} = (\sqrt{3})^2 \times (\sqrt{2})^2$

$= 3 \times 2 = 6$

(12) $\frac{2^4 \times \sqrt{3}^4}{3^4 \times \sqrt{2}^4} = \frac{2^4 \times 3^2}{3^4 \times 2^2} = 2^2 \times 3^{-2}$

$= 4 \times \frac{1}{9} = \frac{4}{9}$

(13) $\frac{3^{x-1} \times 2^{x+1}}{3^{x-1} \times 2^{x-1}} = 2^{x+1-x+1} = 2^2 = 4$

(14) $(\sqrt{3})^{+1} + \left(\frac{1}{\sqrt{3}}\right)^{-4} = 3^2 + 3^2 = 18$

(15) $\left(\frac{9 \times 8}{18}\right)^x = 4^x \quad \therefore 4^x = 4^3$

$\therefore \boxed{x = 3}$

(16) let the width $= x$, Length $= x+5$

$x(x+5) = 36$

$x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

$x = -9$ neglected

or $\boxed{x = 4}$

$\therefore$ the width $= 4$ cm, Length $= 9$ cm

$\therefore P = (4+9) \times 2 = 26$ cm

(17) $\frac{(2^2)^{x+1} \times (3^2)^{2-x}}{2^{2x} \times 3^{2x}} = \frac{2^{2x+2} \times 3^{4-2x}}{2^{2x} \times 3^{2x}}$

$= 2^{2x+2-2x} \times 3^{4-2x-2x}$

$= 2^2 \times 3^{4-4x} = 4 \times 3^{-4} = \frac{4}{81}$

(18) $3^{2x-3} = 3^5$

$\therefore 2x - 3 = 5 \quad \boxed{x = 4}$

No pain, No gain

(19) Let the number is x

$x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4$ or $x = 3$

$\therefore$ the number is -4 or 3

(20) As No. (15)

Second: Geometry

(1) c	(2) b	(3) c	(4) b
(5) a	(6) b	(7) d	(8) b
(9) b	(10) d	(11) c	(12) b
(13) b	(14) d	(15) a	(16) d
(17) b	(18) b	(19) c	(20) a
(21) c	(22) c	(23) a	(24) d
(25) b	(26) a	(27) c	(28) b
(29) c	(30) b	(31) a	(32) a
(33) b	(34) c	(35) c	(36) b
(37) c	(38) b	(39) b	(40) c

Complete:

(1) the same area (2) $m(\angle C)$

(3) the point A (4) 3.5 cm

(5) 45 cm² (6) Proportional, equal in measure

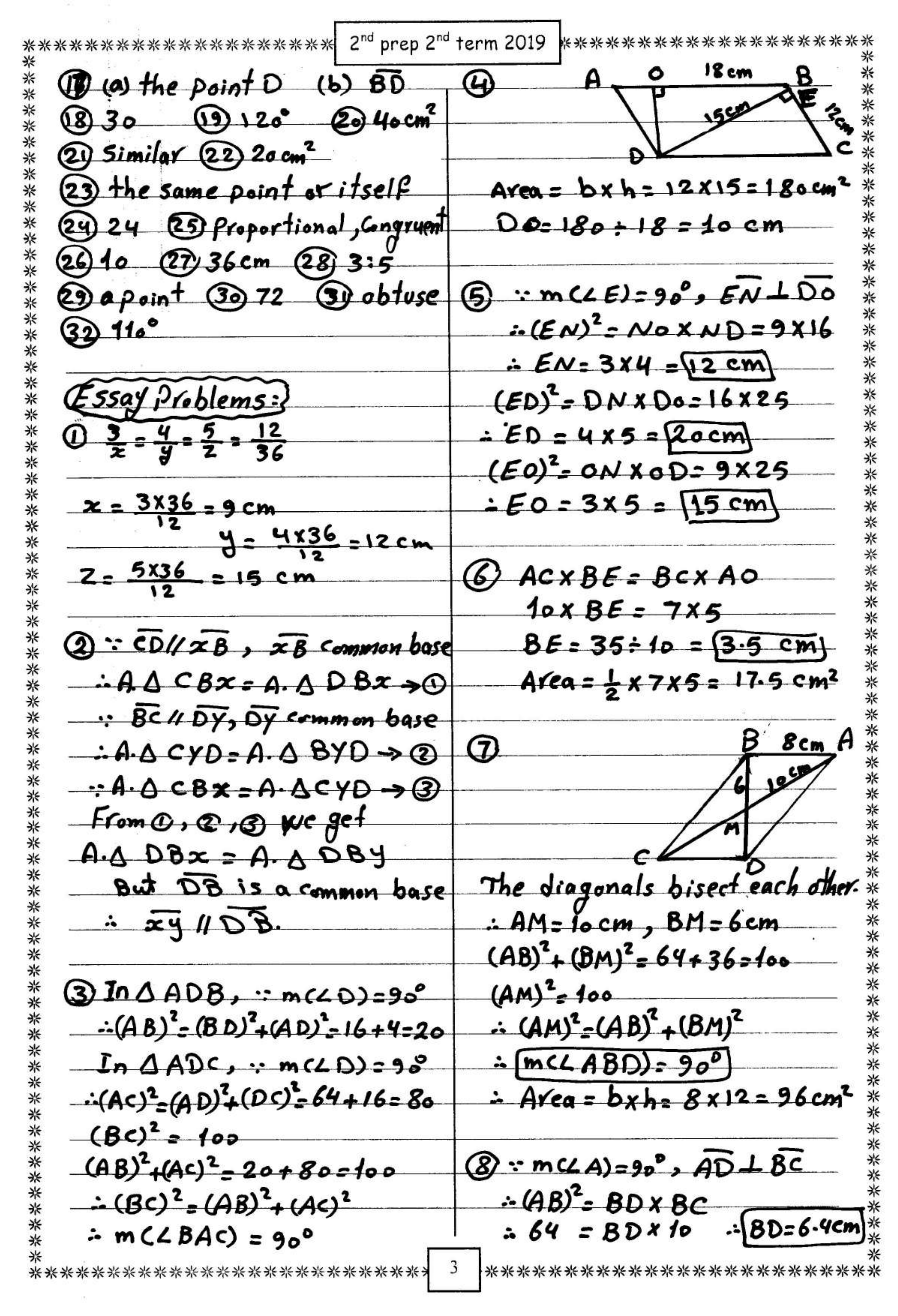
(7) 6 cm (8) B (9) obtuse

(10) base and lie between two Parallel straight line

(11) a Point (12) >

(13) 32 (14) are equal in area

(15) base (16) an obtuse



(17) (a) the point D (b) $\overline{BD}$

(18) 30 (19) $120°$ (20) $40\ cm^2$

(21) Similar (22) $20\ cm^2$

(23) the same point or itself

(24) 24 (25) Proportional, Congruent

(26) 10 (27) 36cm (28) 3:5

(29) a point (30) 72 (31) obtuse

(32) $110°$

Essay Problems:

① $\frac{3}{x} = \frac{4}{y} = \frac{5}{z} = \frac{12}{36}$

$x = \frac{3 \times 36}{12} = 9\ cm$

$y = \frac{4 \times 36}{12} = 12\ cm$

$z = \frac{5 \times 36}{12} = 15\ cm$

② $\because \overline{CD} \parallel \overline{xB}$, $\overline{xB}$ common base

$\therefore A.\Delta CBx = A.\Delta DBx \rightarrow ①$

$\because \overline{BC} \parallel \overline{DY}$, $\overline{DY}$ common base

$\therefore A.\Delta CYD = A.\Delta BYD \rightarrow ②$

$\because A.\Delta CBx = A.\Delta CYD \rightarrow ③$

From ①, ②, ③ we get

$A.\Delta DBx = A.\Delta DBY$

But $\overline{DB}$ is a common base

$\therefore \overline{xy} \parallel \overline{DB}$.

③ In ΔADB, $\because m(\angle D) = 90°$

$\therefore (AB)^2 = (BD)^2 + (AD)^2 = 16 + 4 = 20$

In ΔADC, $\because m(\angle D) = 90°$

$\therefore (AC)^2 = (AD)^2 + (DC)^2 = 64 + 16 = 80$

$(BC)^2 = 100$

$(AB)^2 + (AC)^2 = 20 + 80 = 100$

$\therefore (BC)^2 = (AB)^2 + (AC)^2$

$\therefore m(\angle BAC) = 90°$

④

Area $= b \times h = 12 \times 15 = 180\ cm^2$

$DO = 180 \div 18 = 10\ cm$

⑤ $\because m(\angle E) = 90°$, $\overline{EN} \perp \overline{DO}$

$\therefore (EN)^2 = NO \times ND = 9 \times 16$

$\therefore EN = 3 \times 4 = \boxed{12\ cm}$

$(ED)^2 = DN \times DO = 16 \times 25$

$\therefore ED = 4 \times 5 = \boxed{20\ cm}$

$(EO)^2 = ON \times OD = 9 \times 25$

$\therefore EO = 3 \times 5 = \boxed{15\ cm}$

⑥ $AC \times BE = BC \times AO$

$10 \times BE = 7 \times 5$

$BE = 35 \div 10 = \boxed{3.5\ cm}$

Area $= \frac{1}{2} \times 7 \times 5 = 17.5\ cm^2$

⑦

The diagonals bisect each other.

$\therefore AM = 10\ cm$, $BM = 6\ cm$

$(AB)^2 + (BM)^2 = 64 + 36 = 100$

$(AM)^2 = 100$

$\therefore (AM)^2 = (AB)^2 + (BM)^2$

$\therefore \boxed{m(\angle ABD) = 90°}$

$\therefore$ Area $= b \times h = 8 \times 12 = 96\ cm^2$

⑧ $\because m(\angle A) = 90°$, $\overline{AD} \perp \overline{BC}$

$\therefore (AB)^2 = BD \times BC$

$\therefore 64 = BD \times 10$ $\therefore \boxed{BD = 6.4\ cm}$

(9) In $\triangle ABD$, $\because m(\angle D) = 90°$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = 100$

$\therefore BD = \sqrt{100} = 10cm$

In $\triangle ADC$, $\because m(\angle D) = 90°$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = 324$

$\therefore CD = \sqrt{324} = 18$ cm

$\therefore BC = 10 + 18 = \boxed{28 cm}$

$Area = \frac{1}{2} \times 28 \times 24 = 336 cm^2$

(10) $\because EFD \sim \triangle ABC$

$\therefore \frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED} = \frac{P.\triangle ABC}{P.\triangle EFD}$

$\therefore \frac{4}{EF} = \frac{5}{FD} = \frac{6}{ED} = \frac{15}{60}$

$\therefore EF = \frac{4 \times 60}{15} = 16 cm$

$FD = \frac{5 \times 60}{15} = 20 cm$

$ED = \frac{6 \times 60}{15} = 24 cm$

(11) $\because \triangle ABC \sim \triangle AED$

$\therefore \frac{AB}{AE} = \frac{BC}{ED} = \frac{AC}{AD}$

$\therefore \frac{8}{4} = \frac{8}{ED} = \frac{AC}{3}$

$\therefore ED = \frac{4 \times 8}{8} = \boxed{4 cm}$

$AC = \frac{3 \times 8}{4} = 6 cm$

$\therefore EC = 6 - 4 = \boxed{2 cm}$

(12) $\triangle\triangle$ ADE, ACB

in which $\begin{cases} m(\angle E) = m(\angle B) \\ \angle A \text{ is common} \end{cases}$

$\therefore \triangle ADE \sim \triangle ACB$

$\therefore \frac{AD}{AC} = \frac{AE}{AB}$ $\quad \therefore \frac{3}{AC} = \frac{4.5}{9}$

$\therefore AC = \frac{3 \times 9}{4.5} = 6 cm$ $\quad \boxed{\therefore EC = 1.5 cm}$

(13) In $\triangle ABC$, $\because \overline{AD}$ is a median

$\therefore A.\triangle ABD = A.\triangle ACD \rightarrow (1)$

In $\triangle EBC$, $\because \overline{ED}$ is a median

$\therefore A.\triangle EBD = A.\triangle ECD \rightarrow (2)$

by subtracting (1) - (2)

$\therefore A.\triangle ABE = A.\triangle ACE$.

(14) In $\triangle ABD$, $\because m(\angle B) = 90°$

$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 225$

$\therefore (BD)^2 = \sqrt{225} = \boxed{15 cm}$

In $\triangle BCD$

$(BD)^2 = 225$

$(BC)^2 + (DC)^2 = 81 + 144 = 225$

$\therefore (BD)^2 = (BC)^2 + (DC)^2$

$\therefore m(\angle C) = 90°$

(15) $Area = \frac{5+7}{2} \times 4 = 24 cm^2$

(16) $(BD)^2 = (BC)^2 + (DC)^2 = 625$

$\therefore BD = \sqrt{625} = \boxed{25 cm}$

$(AD)^2 = (BD)^2 - (AB)^2 = 400$

$\therefore AD = \sqrt{400} = \boxed{20 cm}$

The projection of $\overline{AB}$ on $\overleftrightarrow{BD}$ is $\overline{EB}$

$(AB)^2 = EB \times DB$

$225 = EB \times 25$

$\boxed{EB = 9 cm}$

(17) $\frac{3}{5} = \frac{x}{60}$

$x = \frac{3 \times 60}{5} = 36 cm$

(18) $(AC)^2 = (AB)^2 + (BC)^2 = 9 + 16 = 25$

$\therefore AC = \sqrt{25} = \boxed{5\text{ cm}}$

$(AD)^2 = 169$

$(AC)^2 + (DC)^2 = 25 + 144 = 169$

$\therefore (AD)^2 = (AC)^2 + (DC)^2$

$\therefore m(\angle ACD) = 90^\circ$

(19) $(BC)^2 = 8^2 = 64\text{ cm}^2$

$(AB)^2 + (AC)^2 = 36 + 16 = 52\text{ cm}^2$

$\therefore (BC)^2 > (AB)^2 + (AC)^2$

$\therefore \triangle ABC$ is obtuse-angled.

(20) المطلوب الأول هو: $\triangle AED \sim \triangle CEB$

$\triangle\triangle AED, CEB$

in which $\begin{cases} m(\angle A) = m(\angle C) \text{ Alt.} \\ m(\angle D) = m(\angle B) \text{ Alt.} \end{cases}$

$\therefore \triangle AED \sim \triangle CEB$

$\therefore \frac{AE}{CE} = \frac{ED}{EB} = \frac{AD}{CB} = \frac{P.\triangle AED}{P.\triangle CEB}$

$\frac{4}{8} = \frac{9}{P.\triangle CEB}$

$\therefore P.\triangle EBC = \frac{8 \times 9}{4} = 18\text{ cm}$

(21) $\because m(\angle B) = 90^\circ$, $\overline{BD} \perp \overline{AC}$

$\therefore (BD)^2 = DA \times DC = 1.8 \times 3.2$

$\therefore BD = \sqrt{1.8 \times 3.2} = \boxed{2.4\text{ cm}}$

$(AB)^2 = AD \times AC = 1.8 \times 5 = 9$

$\therefore AB = \sqrt{9} = \boxed{3\text{cm}}$

(22) $\because A.\triangle ABE = A.\triangle ACD$

by subtracting $A.\triangle ADE$

$\therefore A.\triangle DEB = A.\triangle DEC$

But $\overline{DE}$ is a common base

$\therefore \overline{DE} \parallel \overline{BC}$

(23) $\because \triangle AXY \sim \triangle ABC$

$\therefore \frac{AX}{AB} = \frac{XY}{BC} = \frac{AY}{AC}$

$\therefore \frac{7}{14} = \frac{XY}{BC} = \frac{6}{AC}$

$\therefore AC = \frac{6 \times 14}{7} = 12\text{ cm}$

$\therefore XC = 12 - 7 = \boxed{5\text{ cm}}$

$\frac{XY}{BC} = \frac{7}{14} = \boxed{\frac{1}{2}}$

(24) $\frac{2x + 3x}{2} = 30$

$5x = 60 \Rightarrow x = 12$

$\therefore$ the 2 bases are: 24 cm, 36 cm

Area $= 30 \times 24 = 720\text{ cm}^2$

(25) $\because \overline{AD} \parallel \overline{BC}$ and $\overline{BC}$ common

$\therefore A.\triangle ABC = A.\triangle DBC$

by subtracting $A.\triangle MBC$

$\therefore A.\triangle AMB = A.\triangle DMC \rightarrow (1)$

$\because \overline{MD}$ is a median in $\triangle EMC$

$\therefore A.\triangle MDE = A.\triangle DMC \rightarrow (2)$

From (1), (2) we get

$A.\triangle AMB = A.\triangle MDE$.

(26) $\because m(\angle A) = 90^\circ$, $\overline{AD} \perp \overline{BC}$

$\therefore (AD)^2 = DC \times DB = 9 \times 16$

$\therefore AD = 3 \times 4 = \boxed{12\text{ cm}}$

$\because (AB)^2 = BD \times BC = 9 \times 25$

$\therefore AB = 3 \times 5 = \boxed{15\text{ cm}}$

$\because (AC)^2 = DC \times BC = 16 \times 25$

$\therefore AC = 4 \times 5 = \boxed{20\text{cm}}$

(27) In ΔBCD, $\because m(\angle C) = 90^\circ$

$\therefore (BD)^2 = (BC)^2 + (DC)^2 = 100$

$\therefore BD = 10$ cm

$\because AB = AD$, E is midpoint of $\overline{BD}$

$\therefore \overline{AE} \perp \overline{BD}$

In ΔABE, $\because m(\angle E) = 90^\circ$

$\therefore (AE)^2 = (AB)^2 - (BE)^2 = 169 - 25$

$= 144$

$\therefore AE = \sqrt{144} = 12$ cm

$\therefore$ A. $\Delta BCD = \frac{1}{2} \times 8 \times 6 = 24\ cm^2$

A. $\Delta ABD = \frac{1}{2} \times 10 \times 12 = 60\ cm^2$

$\therefore$ A. of $ABCD = 24 + 60 = 84\ cm^2$

(28) $BC = 20$ cm

$2AD = 20$ $\therefore AD = 10$ cm

middle base $= \frac{20+10}{2} = 15$ cm

$\therefore h = 180 \div 15 = 12$ cm

(29) As No. (18)

Algebra

Essay

(21)
- $(x^3+8)+(2x^2+4x)$

  $= (x+2)(x^2-2x+4) + 2x(x+2)$

  $= (x+2)(x^2 - 2x + 4 + 2x)$

  $= (x+2)(x^2+4)$

- $(5a^2-1)(5a^2+1)$
- $(x-3)(x^2+3x+9)$
- $(y-8)(y+1)$
- $(5x-3)^2$

(22) $\left(\frac{3}{5}\right)^{x-2} = \left(\frac{3}{5}\right)^3$

$\therefore x - 2 = 3$ $\boxed{\therefore x = 5}$

(23) $\left(\frac{8\times9}{18}\right)^x = 64$

$4^x = 4^3$ $\therefore \boxed{x = 3}$

(24) $\left(\sqrt{\frac{2}{3}}\right)^x = \left(\sqrt{\frac{2}{3}}\right)^4$

$\therefore \boxed{x = 4}$

$\therefore \left(\frac{2}{3}\right)^{x-1} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

(25) S.S. $= \{0, -4, \frac{1}{2}\}$

(26) $\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{2}{5}\right)^3$

$\therefore 2x - 1 = 3$

$\therefore \boxed{x = 2}$

(27) $x^2 + 3x = 28$

$x^2 + 3x - 28 = 0$

$(x+7)(x-4) = 0$

$x = -7$ neglected

or $x = 4$

$\therefore$ the number is 4

(28) $x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$\therefore$ S.S. $= \{3, 5\}$

(29) $\frac{(27)^{-1} \times 27^x \times 8^x}{(2\sqrt{3})^{2x} \times (3\sqrt{2})^{2x}}$

$= (27)^{-1} \times \left(\frac{27\times8}{12\times18}\right)^x = \frac{1}{27} \times 1 = \frac{1}{27}$

(30) $\left(\frac{4\times36}{16\times9}\right)^n = 1^n = 1$

Geometry

Essay:

(30) $\because ABCD \sim XYZL$

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{AD}{XL} = \frac{P1}{P2}$

$\therefore \frac{9}{3} = \frac{29}{P2}$

$P = \frac{3 \times 29}{9} = 9\frac{2}{3}$ cm

(31) $(AB)^2 = (3.5)^2 = 12.25$

$(BC)^2 + (AC)^2 = 6.25 + 9 = 15.25$

$\therefore (AB)^2 < (BC)^2 + (AC)^2$

$\therefore \Delta ABC$ is an acute-angled

(32) $\because m(\angle B) = 90°$, $\overline{BD} \perp \overline{AC}$

$\therefore (AB)^2 = AD \times AC = 9 \times 25$

$\therefore AB = 3 \times 5 = \boxed{15 \text{ cm}}$

$\therefore (BD)^2 = AD \times CD = 9 \times 16$

$\therefore BD = 3 \times 4 = \boxed{12 \text{ cm}}$

(33) $(AC)^2 = (AB)^2 + (BC)^2 = 9 + 16 = 25$

$\therefore AC = 5$ cm

$(AD)^2 = 169$

$(AC)^2 + (DC)^2 = 144 + 25 = 169$

$\therefore (AD)^2 = (AC)^2 + (DC)^2$

$\therefore m(\angle ACD) = 90°$

(34) $\because \overline{AD} // \overline{BC}$, $\overline{BC}$ common base

$\therefore A.\Delta ABC = A.\Delta DBC$

by subtracting $A.\Delta MBC$

$\therefore$ A. of $\Delta AMB = A.\Delta DMC \to (1)$

$\because AX = DY$, M common vertex

$\therefore A.\Delta AXM = A.\Delta DYM \to (2)$

From (1), (2) we get

A. of ABMX = A. of DCMY.

(35) $\frac{1}{3} = \frac{20}{x}$

$x = \frac{3 \times 20}{1} = 60$ cm

(36) $\Delta\Delta$ ADE, ABC

in which $\begin{cases} m(\angle D) = m(\angle B) \text{ corres.} \\ m(\angle E) = m(\angle C) \text{ corres.} \\ \angle A \text{ common} \end{cases}$

$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$ $\therefore \frac{8}{10} = \frac{5}{AC}$

$\therefore AC = \frac{5 \times 10}{8} = 6.25$ cm

$\therefore x = 6.25 - 5 = \boxed{1.25 \text{ cm}}$

(37) $\because \Delta ABC \sim \Delta ADE$

$\therefore \frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$ $\therefore \frac{14}{7} = \frac{AC}{6} = \frac{BC}{DE}$

$\therefore AC = \frac{6 \times 14}{7} = 12$ cm

$\therefore DC = 12 - 7 = \boxed{5 \text{ cm}}$

$\therefore \frac{DE}{BC} = \frac{7}{14} = \frac{1}{2}$

(38) $\because \overline{AD} // \overline{BC}$, $\overline{AD}$ common base

$\therefore A.\Delta ADB = A.\Delta ADC$

by subtracting $A.\Delta ADM$

$\therefore A.\Delta AMB = A.\Delta DMC \to (1)$

$\because xB = CY$, M is common vertex

$\therefore A.\Delta MBx = A.\Delta MCY \to (2)$

From (1), (2) we get

A. ABXM = A. DCYM.







Part (1)

(1) Complete the following:

1) The area of the triangle whose base length 10cm and height 6cm equals ……… cm^2.

2) Two triangles which have the same base and their vertices opposite to this base on a straight line parallel to the base are …….. in area.

3) The area of the rhombus whose diagonals 12 cm, 8 cm equals ………….. cm^2.

4) The median of a triangle divide it into two triangle …..... in the area,

5) The area of trapezium whose parallel base 6 cm, 10 cm and height 5 cm. equals ………….

6) If two triangles have equal areas and drawn on the same base and in one side of it then …………

7) Surface of two parallelograms with common base and between two parallel lines ……………..

8) The median of a triangle divides its surface into ……..

9) Area of the parallelogram equals ………

10) Triangles of equal bases in length and lying between two parallel lines are equal in ……………

11) The area of the rhombus whose diagonals X cm, Y cm is …….

12) The area of the right angled triangle whose sides length of the right angle are 6 cm , 8 cm equals ……..

13) The area of the trapezium whose middle base 9 cm and height 6 cm equals ………

14) The measure of base angles of an isosceles trapezium are …..

15) The lengths of two adjacent sides in a parallelogram are 9 cm, 6 cm and the smallest height is 4cm then the length of the other height is …….

16) The height of trapezium whose parallel base are 5 cm, 7 cm and area of 42 cm^2 is …………….

17) The area of rhombus whose perimeter is 20 cm and height 4 cm = …………

18) The length of the diagonal of a square of area 50 cm^2 equals …. cm .

19) The length of side of a square whose area equals the area of a rectangle with dimensions 9 cm , 16 cm = …….

20) The length of the middle base of a trapezium whose area = 30 cm^2 and height 5 cm equals ………..

(2) Choose the correct answer:-

1) The length of the base of a triangle whose area 30 cm^2 and height 6 cm….

a) 5 b) 10 c) 15 d) 20

2) The length of the two adjacent sides in a parallelogram are 7 cm, 5 cm and the length of its smallest height is 4 cm then the area of the parallelogram equals ……… cm^2.

a) 35 b) 25 c) 28 d) 49

3) The area of trapezium whose middle base length is 10 cm and height 8 cm equals ……… cm^2.

a) 80 b) 60 c) 40 d) 20





4) The quadrilateral whose area equals half square of its diagonal is …………

a) parallelogram b) rectangle c) rhombus d) square

5) The diagonals of an isosceles trapezium ………..

a) congruent b) perpendicular

c) bisect each other d) parallel

6) The area of rhombus whose diagonals length are 6 cm, 8 cm equals…..

a) 2 cm^2 b) 14 cm^2 d) 24 cm^2 d) 48 cm^2

7) The ratio between area of parallelogram and area of triangle if they have a common base and including between two parallel lines equals ……..

a) 1 : 2 b) 1 : 3 d) 2 : 1 d) 2 : 3

8) If the area of a square 18 cm^2 then length of its diagonal is …

a) 36 b) 12 c) 9 d) 6

9) If two triangles area equal in area and drawn on same base and in one side of it then their vertices lie on a straight line.

a) perpendicular to this base. b) bisect this base

c) parallel to this base d) intersects the base.

10) The quadrilateral whose area equals the square of its side length is…

a) parallelogram b) rectangle

c) rhombus d) square

11) The area of the rectangle whose dimensions 5 cm, 4 cm is ….

a) 9 cm^2 b) 10 cm^2 c) 20 cm^2 d) 40 cm^2

12) The side length of a square whose area equals the area of a parallelogram of base length 8 cm and corresponding height 4.5cm equals…...

a) 6 cm　　b) 13 cm　　c) 18 cm　　d) 36 cm

13) The median of a triangle divides its surface into two triangles ….

a) congruent　　b) equals in area

c) isosceles　　d) right angles

14) The perimeter of the square whose area 81 cm^2 = ….. cm.

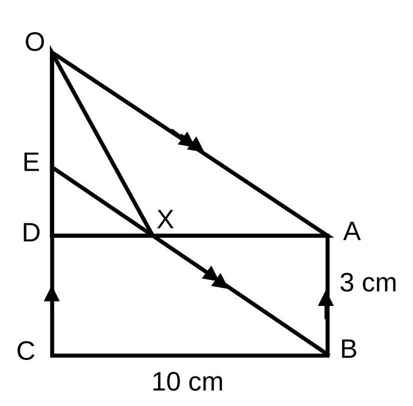
a) 24　　b) 8　　c) 9　　d) 36

15) If the area of a rhombus is 24 cm^2 and the length of one of its diagonal is 6 cm then the length of the other diagonal is …….

a) 4 cm　　b) 8 cm　　c) 10 cm　　d) 12 cm

(3) Essay Questions:-

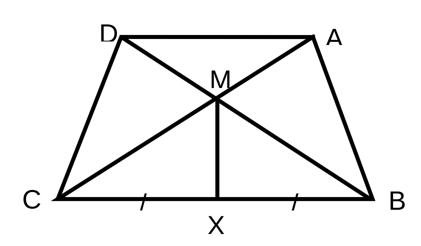
(1) In opposite figure :



ABCD is a rectangle, ABEO is a parallelogram, AB = 3 cm, BC = 10 cm

Find with proof: the area of Δ AXO

(2) In the opposite figure:



$\overline{AD} // \overline{BC}$, X midpoint of $\overline{BC}$ prove that:

(i) Area of Δ AMB = area of Δ DMC

(ii) Area of shape ABXM = area of shape DCXM



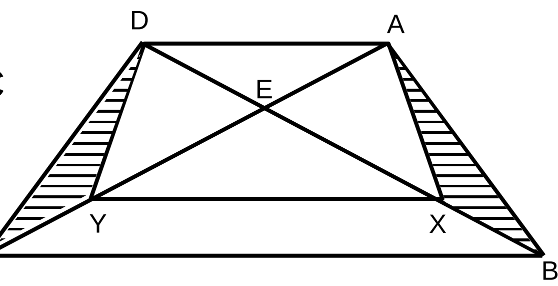


(3) The area of a trapezium is 88 cm², its height is 8 cm and the length of one of the two parallel base 10 cm, find the length of the other base.

<u>**(4) In the opposite figure:**</u>

$\overline{AD} \parallel \overline{BC}$ area of Δ AXB = area of Δ DYC

Prove that: $\overline{XY} \parallel \overline{AD}$



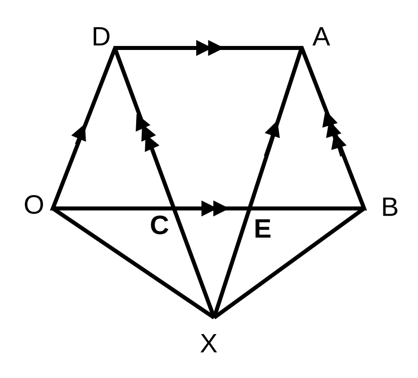
<u>**(5) In the opposite figure:**</u>

ABCD , AEOD area two parallelograms

$\overrightarrow{AE} \cap \overrightarrow{DC} = \{X\}$

Prove that

Area of Δ ABX equals area of Δ DOX

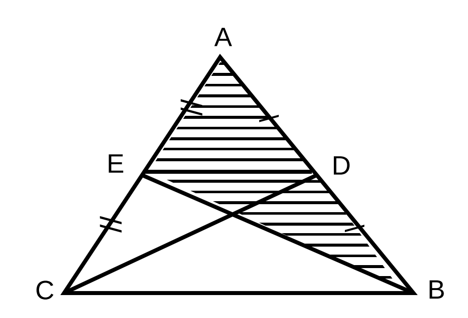


(6) Two pieces of land have equal areas, one of them has the shape of a square and the other has the shape of trapezium with two parallel bases of lengths 7 m, 11 m and height of 4m find the perimeter of the square land.

<u>**(7) In the opposite figure**</u>

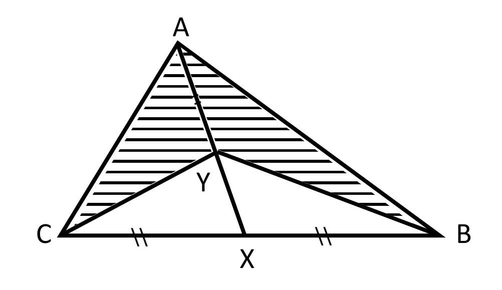
If area of (Δ ADC) = are of (Δ AEB)

Prove that $\overline{DE} \parallel \overline{BC}$







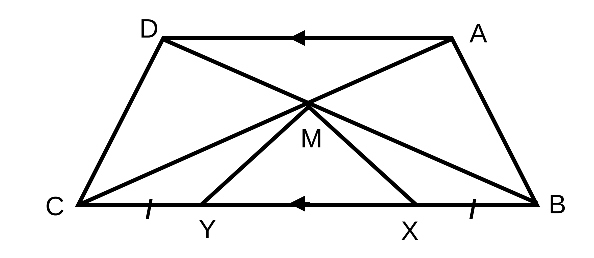


(8) In the opposite figure:

$\overline{AX}$ is a median in Δ ABC

, $Y \in \overline{AX}$, $\overline{BY}$, $\overline{CY}$ are drawn prove that

area of (Δ ABY) = area of (Δ ACY)



(9) In the opposite figure:

$\overline{AD} // \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

$X , Y \in \overline{BC}$ such that BX = CY

Prove that:

area of shape ABXM = area of shape DCYM

(10) ABCD is a parallelogram in which $\overline{DE} \perp \overline{BC}$, $\overline{DO} \perp \overline{AB}$

if AB = 4 cm, BC = 6 cm , DE = 3 cm find the length of $\overline{DO}$

Part (2)

First : Complete the following:

1) If $\overline{AB} \perp \overline{BC}$ then the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ……..

2) In Δ ABC if $(AB)^2 = (BC)^2 + (AC)^2$ then m ($\angle$ …....) = 90°

3) The two polygons are similar to a third are ………..

4) The two triangles are similar if its corresponding angles are ……..
in measure.

5) ABC is a right angled triangle at B in which AB = 5 cm, BC = 12 cm
then AC = …….. cm.

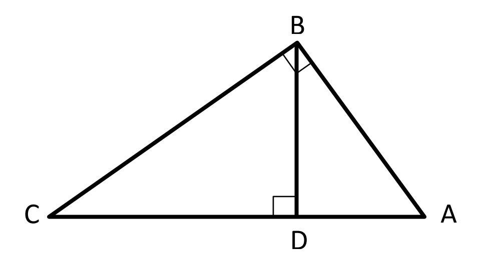
6) The projection of a point which belongs to a straight line on this line
is ………....

7) In Δ ABC if $(AC)^2 + (AB)^2 < (BC)^2$ then angle A is ……….

8) In Δ XYZ if $(ZX)^2 + (YZ)^2 > (XY)^2$ then angle Z is ……..….

9) In the opposite figure:

Δ ABC is right angle triangle at B, $\overline{BD} \perp \overline{AC}$



a) The projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ is

b) $(AB)^2$ = AD × ………

c) $(BD)^2$ = AD × ………

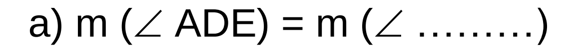
d) $(BC)^2$ = CD × ………

e) Δ ABC ~ Δ ……… ~ Δ ………

10) In the opposite figure:

If Δ AED ~ Δ ABC, AD = 3 cm, AE = 4 cm,

BC = 10 cm , BD = 5 cm then

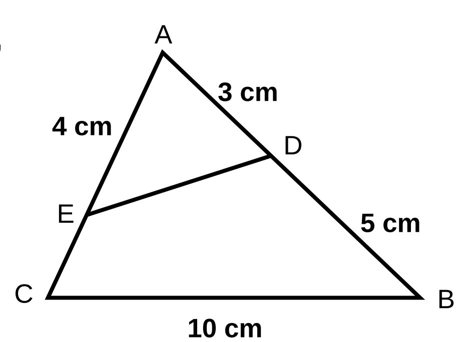


a) m (∠ ADE) = m (∠ ....)

b) m (∠ BAC) = m (∠ ....)

c) DE = ... cm

d) ED = ... cm



11) The area of a rectangle whose length of one of its dimensions = 12 cm, its diagonal = 13 cm equal ....

12) The triangle of side length 3 cm, 4 cm, 5 cm is .... angled triangle.

13) Two triangles are similar one of them has sides length 9 cm, 12 cm, 16 cm and the perimeter of the other 148 cm then side lengths of the other triangle are .... , ... , ...

Second: Choose the correct answer:

1) If Δ ABC ~ Δ DEO, AB = $\frac{1}{4}$ DE then the perimeter of Δ ABC equals .... the perimeter of Δ DEO.

a) 4 b) 2 c) $\frac{1}{2}$ d) $\frac{1}{4}$

2) The length of the projection of a given line segment .... the length of the original line segment.

a) $\geq$ b) $>$ c) $\leq$ d) $<$

3) ABC is an obtuse angle triangle at A in which AB = 5 cm, BC = 8 cm then AC = ... cm

a) 5 b) 7 c) 8 d) 13

4) The triangle whose sides length are 3 cm, 4 cm, 5 cm its area = … cm^2

a) 12 b)10 c) 8 d) 6

5) If the ratio of enlargement between two similar triangles equals …………. then the two triangles are congruent.

a) 1 b) 2 c) 0.5 d) 0.25

6) Δ ABC in which $(AC)^2 = (BC)^2 - (AB)^2$ then angle A is …………

a) acute b) right c) obtuse d) straight

7) The triangle whose sides length are 5 cm, 12 cm, 13 cm its area = ………… cm^2

a) 30 b) 32.5 c) 78 d) 144

8) Δ ABC is obtuse angle triangle at B and AB = 3 cm, BC = 5 cm then AC = …………

a) 8 cm b) 7 cm c) 15 cm d) 4 cm

9) In the two similar polygons their corresponding angles are ……… in measure.

a) equal b) difference c) proportional d) alternatives

10) The perpendicular segment drawn from the right angle of a triangle to the hypotenuse divides it to two triangles.

a) obtuse angle b) acute angle

c) equal's sides triangle d) similar

11) ABC is a triangle in which $\overline{AD} \perp \overline{BC}$ then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is …………

a) $\overline{BD}$ b) $\overline{DC}$ c) $\overline{AC}$ d) $\overline{AB}$

12) Δ ABC in which $(AB)^2 + (BC)^2 < (AC)^2$ then $\angle$ B is …………

a) acute b) right c) obtuse d) reflex





13) The diagonal of a square whose area 50 cm^2 equals …………

a) 10 cm　　b) 20 cm　　c) 30 cm　　d) 40 cm

14) Δ ABC in which $(AB)^2 = (AC)^2 + (BC)^2$, m ($\angle$ B) = 40° then m ($\angle$ A) = …………

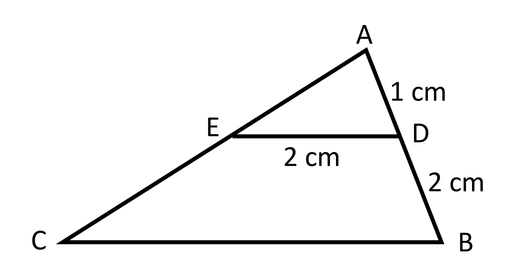
a) 40°　　b) 50°　　c) 90°　　d) 130°

15) In the opposite figure:

If Δ ADE ~ Δ ABC then the length of $\overline{BC}$ in cm equals ……….

a) 3　　b) 4

c) 6　　d) 8



Third: Essay question:

(1) Determine the type of the angle B in ΔABC in each of the following:

a) AB = 7 cm , BC = 12 cm , AC = 8 cm

b) AB = 5 cm , BC = 8 cm , AC = 11 cm

c) AB = 6 cm , BC = 3.6 cm , AC = 4.6 cm

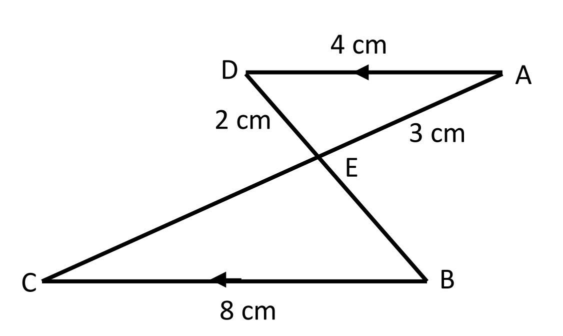
(2) In the opposite figure:

$\overline{AD}$ // $\overline{BC}$, AD = 4 cm, BC = 8 cm ,

AE = 3 cm, ED = 2 cm

i) Prove that Δ AED ~ Δ CEB

ii) Find the perimeter of Δ EBC







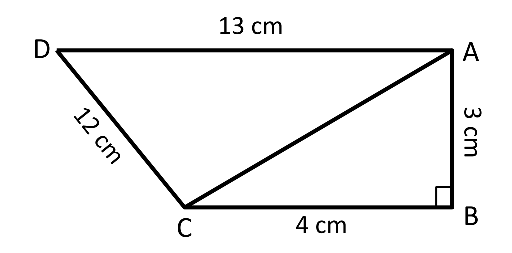
(3) In the opposite figure:

AB = 3 cm , BC = 4 cm ,

AD = 13 cm, CD = 12 cm

m (∠ B) = 90°

Prove that m (∠ ACD) = 90°



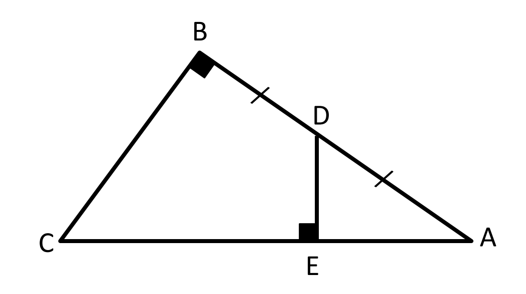
(4) In the opposite figure:

ABC is right angle triangle at B,

D is the midpoint

of $\overline{AB}$, $\overline{DE} \perp \overline{AC}$, AB = 8 cm, BC = 6 cm

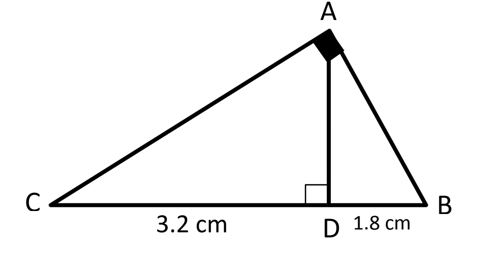
Find the length of $\overline{DE}$



(5) In the opposite figure:

ED = 1.8 cm, DC = 3.2 cm

Find the lengths of each $\overline{AC}$, $\overline{AD}$



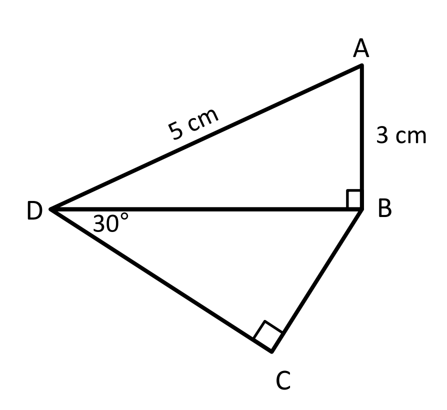
(6) In the opposite figure:

ABCD is quadrilateral in which

m (∠ ABD) = 90°, m (∠ BCD) = 90°,

m (∠ BDC) = 30°,

AB = 3 cm , AD = 5 cm find $\overline{BC}$







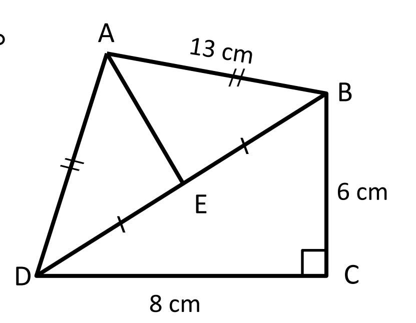
(7) In the opposite figure:

ABCD is a quadrilateral in which m (∠ C) = 90°

AB = AD = 13 cm, BC = 6 cm, CD = 8 cm

E is midpoint of $\overline{BD}$

Find the area of the shape ABCD



(8) In the opposite figure:

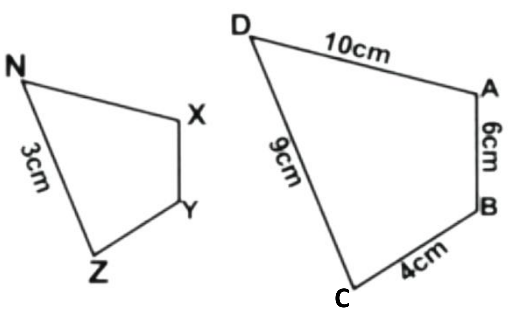
The polygon ABCD

is similar to the polygon XYZN ,

AB = 6 cm , BC = 4 cm ,

CD = 9 cm , DA = 10 cm

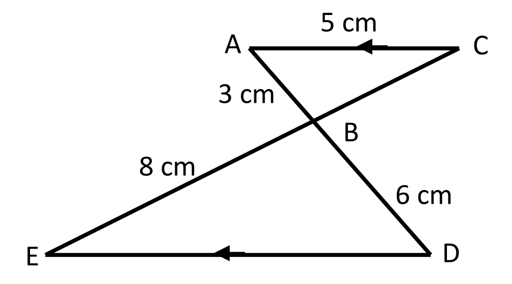
, ZN = 3 cm find the lengths of $\overline{XY}$, $\overline{YZ}$, $\overline{XN}$



(9) In the opposite figure:

i) Prove that Δ ABC is similar Δ DBE

ii) Find the length of $\overline{BC}$, $\overline{DE}$



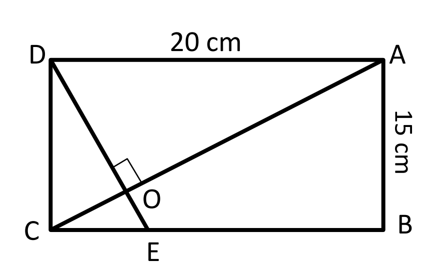
(10) In the opposite figure:

ABCD is a rectangle $\overline{DE} \perp \overline{AC}$

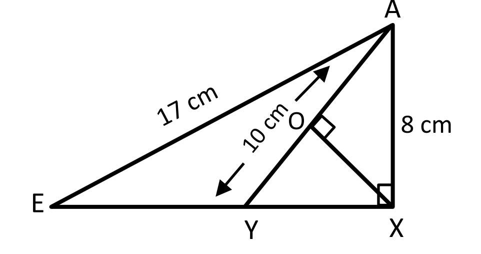
, DE intersect AC at O and intersect BC at E

If AB = 15 cm, AD = 20 cm

Find the lengths of each $\overline{AO}$, $\overline{CE}$



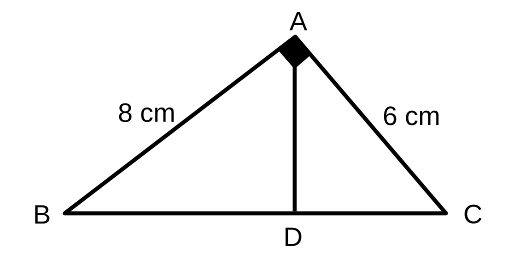
(11) In the opposite figure:



i) Find the length of projection of $\overline{AY}$ on $\overleftrightarrow{XE}$

ii) Find the length of $\overline{XO}$, $\overline{AO}$

(12) In the opposite figure:



Δ DBA ~ Δ ABC , m ($\angle$ BAC) = 90°

Prove that: $\overline{AD} \perp \overline{BC}$

Find BD if AB = 8 cm , AC = 6 cm

(13) A piece of land has a rectangle shape whose length twice its width and its area 200 meter square is drawn by a scale 1:200 find the dimensions of this land at the drawing.

Model Answers

Part (1)

(1) Complete:

1) 30 cm^2

2) equal.

3) 48 cm^2.

4) equal.

5) $\frac{1}{2}$ (6 + 10) × 5 = 40 cm^2

6) Their vertices lie on a straight line parallel to this base.

7) one is carrying this base are equal in area.

8) Two triangular surface equal in area.

9) the length of the base X its corresponding height.

10) Area.

11) $\frac{1}{2}$ XY cm^2.

12) $\frac{1}{2}$ × 6 × 8 = 24 cm^2

13) 9 × 6 = 54 cm^2

14) equal in measure

15) 6 cm.

16) the middle base = $\frac{1}{2}$ (5 + 7) = 6 cm

H = 42 ÷ 6 = 7 cm.

17) b = 20 ÷ 4 = 5 cm

A = 5 × 4 = 20 cm^2

18) 10 cm.

19) A. of rectangle = 9 × 16 = 144 cm^2

S. of square = $\sqrt{144}$ = 12cm.

20) 30 ÷ 5 = 6cm.

(2) Choose the correct answer:-

1) (b)	2) (c)	3) (a)	4) (d)
5) (a)	6) (c)	7) (c)	8) (d)
9) (c)	10) (d)	11) (c)	12) (d)
13) (b)	14) (d)	15) (b)	

(3)

(1) Proof: ∵ ABCD is a rectangle, ABEO is a parallelogram

▭ ABCD , ▱ ABEO have common base $\overline{AB}$

∴ Area of ▭ ABCD = Area of ▱ ABEO

∵ AB = 3 cm , BC = 10 cm

∴ Area of ▭ ABCD = 3 × 10 = 30 cm^2

∴ Area of ▱ ABEO = 30 cm^2

∵ In Δ AXO , ▱ ABEO have common base $\overline{AO}$

, $\overline{AO}$ // $\overline{BE}$

∴ Area of Δ AXO = $\frac{1}{2}$ area of ▱ ABEO

= $\frac{1}{2}$ x 30 = 15 cm

(2) Proof: ∵ $\overline{AD}$ // $\overline{BC}$

In Δ ACD , Δ ADB have common base $\overline{AD}$

∴ Area of Δ ACD = Area of Δ ADB (1)

subtracting A. of Δ AMD from (1)

∴ Area of Δ DMC = Area of Δ AMB (2)

∵ X midpoint of $\overline{BC}$

∴ Area of Δ MXC = Area of Δ MXB (3)

Adding (2) & (3)

∴ Area of the shape DCXM = Area of the shape ABXM





(3) Area of trapezium = $\frac{1}{2}$ (b_1 + b_2) x h

$88 = \frac{1}{2}(10 + b_2) \times 8$

$b_2 = 12$ cm

(4) ∵ $\overline{AD}$ // $\overline{BC}$

In Δ ADB , Δ ADC have common base $\overline{AD}$

∴ Area of Δ ADB = Area of Δ ADC (1)

∵ Area of Δ AXB = Area of Δ DYC (2)

subtracting (2) from (1)

∴ Area of Δ ADX = Area of Δ AYD (3)

have a common base $\overline{AD}$

∴ $\overline{XY}$ // $\overline{AD}$

(5) ∵ ABCD , AEOD are two parallelogram

, $\overline{AD}$ is a common base

∴ Area of ▱ ABCD = Area of ▱ AEOD (1)

subtracting Area of the figure AECD from (1)

∴ Area of Δ ABE = Area of Δ DCO (2)

∴ OC = EB

∵ in Δ XCO , Δ XEB have common vertex X

, EB = CO

∴ Area of Δ XBE = Area of Δ XCO (3)

Adding (2) & (3)

∴ Area of Δ ABX = Area of Δ DOX





(6) Area of trapezium = $\frac{1}{2}$ (b_1 + b_2) x h

= $\frac{1}{2}$ (7 + 11) x 4

= 36 cm^3.

Area of square = 36 cm^3.

S = $\sqrt{36}$ = 6 cm.

Perimeter of square = 6 x 4 = 24 cm^2

(7) Proof: ∵ Area of Δ ADC = Area of Δ AEB

subtracting Area of Δ ADE from both side

∴ Area of Δ EDC = Area of Δ DEB

, $\overline{ED}$ is a common base

∴ $\overline{ED}$ // $\overline{BC}$

(8) Proof: ∵ In Δ ABC

X is midpoint

∴ A. of Δ ABX = A. of Δ AXC (1)

∵ In Δ YBC

X is midpoint

∴ A. of Δ YBX = A. of Δ YXC (2)

subtracting (2) from (1)

∴ A. of Δ ABY = A. of Δ ACY .





(9) Proof: ∵ In Δ ABD , Δ ACD

$\overline{AD}$ // $\overline{BC}$, $\overline{AD}$ is a common base .

∴ Area of Δ ABD = Area of Δ ADC (1)

By subtracting Area of Δ AMD from both side

∴ Area of Δ AMB = Area of Δ DMC (2)

∵ Δ MXB , Δ MYC

M is a common vertex, XB = YC

∴ Area of Δ MXB = A. of Δ MYC (3)

Adding (2) & (3)

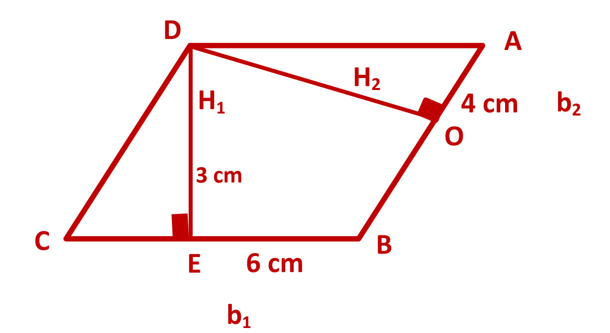
∴ Area of shape ABXM = Area of shape DCYM

(10) Area of parallelogram

$= b_1 \times h_1 = 3 \times 6 = 18\ cm^2$

$A = b_2 \times h_2 = 4 \times h_2 = 18\ cm^2$

h_2 (DO) = 18 ÷ 4 = 4.5 cm







Part (2)

First: Complete:

1) $\overline{BC}$ 2) (∠ C) 3) similar

4) equal 5) 13 cm 6) the same point

7) obtuse 8) acute

9) a) $\overline{AC}$ b) AC c) DC d) $\overline{CA}$ e) Δ ADB - Δ BDC

10) a) m (∠ ACB) b) m (∠ EAD) c) 5 cm d) 2 cm

11) 60 cm^2 12) right

13) 36 cm , 48 cm , 64 cm

Second: Choose:

1) d	2) c	3) a	4) d	5) a
6) b	7) a	8) b	9) a	10) d
11) a	12) a	13) a	14) b	15) c

Third: Essay Question

(1) a) obtuse b) obtuse c) obtuse

(2) ∵ $\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ & $\overline{DB}$ are transversals

∴ m (∠ D) = m (∠ B)

m (∠ A) = m (∠ C) alternate angles → (1)

∵ $\overleftrightarrow{DB} \cap \overleftrightarrow{AC}$ = { E }

∴ m (∠ DEA) = m (∠ BEC) V.O.A → (2)

From (1) & (2)

∴ Δ ADE ~ Δ CBE

$$\therefore \frac{AD}{CB} = \frac{DE}{BE} = \frac{AE}{CE} = \frac{P.of\,\Delta\,ADE}{P.of\,\Delta\,CBE}$$

$\therefore \frac{4}{8} = \frac{2}{BE} = \frac{3}{CE} = \frac{4+2+3}{P.of\,\Delta\,CBE}$

P. of Δ CBE = $\frac{9 \times 8}{4}$ = 18 cm

(3) In Δ ABC: ∵ m (∠ B) = 90°

∴ $(AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras)

AC = $\sqrt{(3)^2 + (4)^2}$ = 5 cm

In Δ ACD

∵ $(AD)^2 = (13)^2 = 169$,

$(AC)^2 = 25$, $(CD)^2 = 144$

∴ $(AD)^2 = (AC)^2 + (CD)^2$

∴ m (∠ ACD) = 90° (converse of Pythagoras theory)

(4) In Δ ABC: ∵ (∠ B) = 90°

∴ $(AC)^2 = (AB)^2 + (BC)^2 = 64 + 36 = 100$

∴ AC = 10 cm

, ∵ D is the midpoint of $\overline{AB}$

∴ AD = DB = 4 cm

In ΔΔ AED , ABC

m (∠ AED) = m (∠ B) = 90° (given)

, ∠ A is common

∴ m (∠ ADE) = m (∠ ACB)

∴ Δ AED ~ Δ ABC

$\therefore \frac{DE}{CB} = \frac{AD}{AC}$, $\therefore \frac{DE}{6} = \frac{4}{10}$

∴ DE = $\frac{6 \times 4}{10}$ = 2.4 cm

(5) In Δ ABC :

$\because$ m ($\angle$ A) = 90° , $\overline{AD} \perp \overline{CB}$

$\therefore$ $(AC)^2$ = CD × CB = 3.2 × 5 = 16 (Euclidean theorem)

AC = 4 cm

$(AD)^2$ = DB × DC = 1.8 × 3.2 = 5.76

AD = 2.4 cm

(6) In Δ ABD: $\because$ m ($\angle$ B) = 90°

$\therefore$ (BD) = $\sqrt{(5)^2 - (3)^2}$ = 4 cm (Pythagoras theorem)

In Δ BCD: $\because$ m ($\angle$ C) = 90° , m ($\angle$ CDB) = 30°

$\therefore$ CB = $\frac{1}{2}$ BD = $\frac{1}{2}$ × 4 = 2 cm

(7) In Δ BCD: $\because$ m ($\angle$ C) = 90°

$\therefore$ BD = $\sqrt{(BC)^2 + (CD)^2}$ = $\sqrt{(6)^2 + (8)^2}$ = 10 cm (Pythagoras Theorem)

In Δ ABD: E is a midpoint of $\overline{BD}$, AB = AD

$\therefore$ AE $\perp$ BD , EB = 5 cm

$\therefore$ AE = $\sqrt{(AB)^2 - (EB)^2}$ = $\sqrt{(13)^2 - (5)^2}$ = 12 cm

$\because$ The area of the quadrilateral ABCD =

Area of Δ BCD + Area of Δ ABD

$\therefore$ Area = $\frac{1}{2}$ × DC × BC + $\frac{1}{2}$ × BD × AE

= $\frac{1}{2}$ × 8 × 6 + $\frac{1}{2}$ × 10 × 12 = 24 + 60 = 84 cm^2

(8) $\because$ Polygon ABCD ~ Polygon XYZN

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZN} = \frac{AD}{XN}$

$\frac{6}{XY} = \frac{4}{YZ} = \frac{9}{ZN} = \frac{10}{XN}$

XY = 2 cm , YZ = $1\frac{1}{3}$ cm , XN = $3\frac{1}{3}$ cm





(9) ∵ $\overline{AC}$ // ED , $\overleftrightarrow{AD}$ & $\overleftrightarrow{CE}$ are transversals

∴ m (∠ A) = m (∠ D)

m (∠ C) = m (∠ E) alternate angles → (1)

∵ $\overleftrightarrow{AD} \cap \overleftrightarrow{CE}$ = { B } , ∴ m (∠ ABC) = m (∠ EBD) V.O.A → (2)

From (1) & (2)

∴ Δ ABC ~ Δ DBE

∴ $\frac{AB}{DB} = \frac{BC}{BE} = \frac{CA}{ED} = \frac{3}{6} = \frac{BC}{8} = \frac{5}{ED}$,

BC = 4 cm , ED = 10 cm

(10) In Δ ABC: ∵ (∠ B) = 90°

∴ AC = $\sqrt{(AB)^2 + (BC)^2} = \sqrt{(15)^2 + (20)^2}$ = 25 cm (Pythagoras)

In Δ ADC: ∵ (∠ D) = 90°

∴ $(DA)^2$ = AO × AC (Euclidean Theorem)

∴ AO = $\frac{(20)^2}{25}$ = 16 cm

∴ DO = $\frac{DA \times DC}{AC} = \frac{20 \times 15}{25}$ = 12 cm

∵ Δ DCE is right angled at C , $\overline{CO} \perp \overline{DE}$

∴ $(CD)^2$ = DO × DE → DE = $\frac{(15)^2}{12}$ = 18.75 cm

OE = 18.75 - 12 = 6.75 cm

$(CE)^2$ = EO × ED = 6.75 × 18.75 = 126.5625 cm^2

CE = 11.25 cm

(11) $\because$ $\overline{XY}$ is the projection of $\overline{AY}$ on $\overleftrightarrow{XE}$, Δ AXY is right angled

$\therefore$ $(XY)^2 = (AY)^2 - (AX)^2 = 100 - 64 = 36$, XY = 6 cm

$\because$ $\overline{XD} \perp \overline{AY}$, $XO = \frac{AX \times XY}{AY} = \frac{6 \times 8}{10} = 4.8$ cm

$(AX)^2 = AF \times AY$, AF = 6.4 cm

(12) $\because$ Δ ABC is right angled at A , $\therefore$ $BC = \sqrt{8^2 + 6^2} = 10$ cm

$\because$ Δ DBA ~ Δ ABC , $\therefore$ m ($\angle$ BDA) = m ($\angle$ BAC) = 90°

$\therefore$ $\overline{AD} \perp \overline{BC}$, $\therefore$ $(BA)^2 = BD \times BC$, $BD = \frac{64}{10} = 6.4$ cm

(13) Let the real length be = 2x , width = x

$A = L \times w = 2x \times x = 2x^2 = 200$ m $\rightarrow$ x = 10 cm , 2x = 20 m

Length in drawing = $\frac{2000 \times 1}{200} = 10$ cm D.L : R.L

Width in drawing = $\frac{1000 \times 1}{200} = 5$ cm 1 : 200